Screening mechanisms and the role of gravity in cosmology

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Parameterized Tests of General Relativity
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29th October 2018

Plan of the day

Lecture 1: From cosmology to black holes

- What cosmologists care about
- Screening mechanisms
- Model building tool-kits
- Where we stand now

Plan of the day

Lecture 2: What I think is interesting

- Strong field vs. cosmology vs. theory
- Outstanding problems this community can resolve
- Best targets for joint efforts
- Possible ideas

Outline

1. Motivation: dark energy
2. Why we need to screen
3. Principles of screening
4. Chameleon mechanism
5. Symmetron mechanism
6. Unified description of screening
7. Vainshtein mechanism and galileons
8. Horndeski and beyond
9. Vainshtein breaking
10. Cosmological gravity after GW170817

Living Rev Relativ (2018) 21:1
https://doi.org/10.1007/s41114-018-0011-x

CrossMark

REVIEW ARTICLE

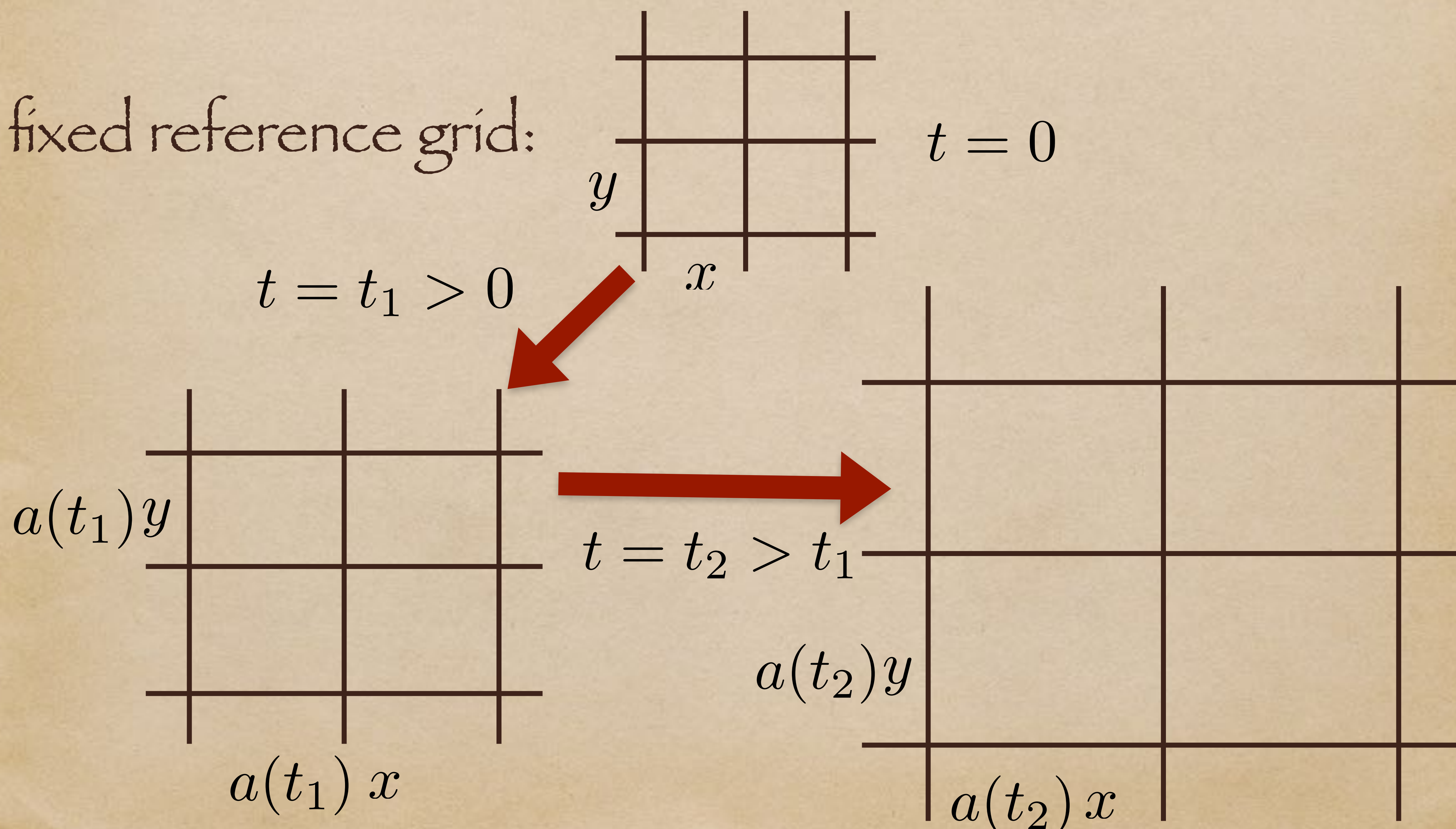
Tests of chameleon gravity

Clare Burrage[1] · Jeremy Sakstein[2]

arXiv: 1709.09071

1) Dark energy

Cosmology 101: how do we describe an expanding universe?



Cosmology 101

Friedmann equation: $$3M_{\rm pl}^2 \left(\frac{\dot{a}}{a}\right)^2 = \sum_i \rho_i$$

density of particle species i

Acceleration equation: $$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}\sum_i (\rho_i + 3P_i)$$

pressure of particle species i

What stuff can we put in?

Equation of state: $P = w\rho$

Speed of sound: $w = c_s^2 = \dfrac{\partial P}{\partial \rho} = w$

E.g.

- Matter/Dark matter — w=0
- Radiation — w=1/3

What does this do?

Acceleration equation: $$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}\sum_i (\rho_i + 3P_i)$$

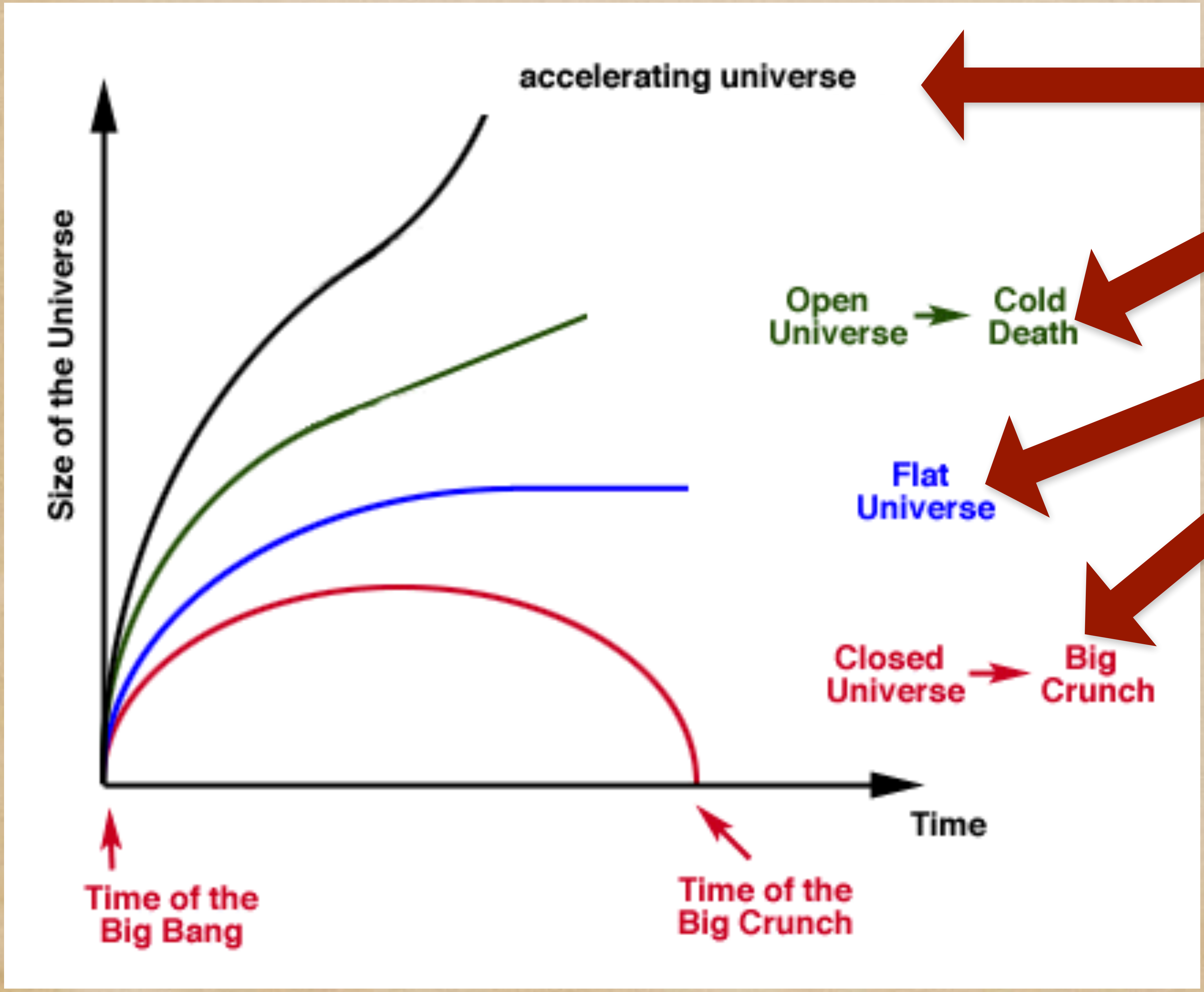
Equation of state: $$P = w\rho$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(1+3w)\,\rho$$

Deceleration unless $w < -1/3 \Rightarrow c_s^2 < -1/3$

Why? Simple: gravity pulls things together!

Possible universes

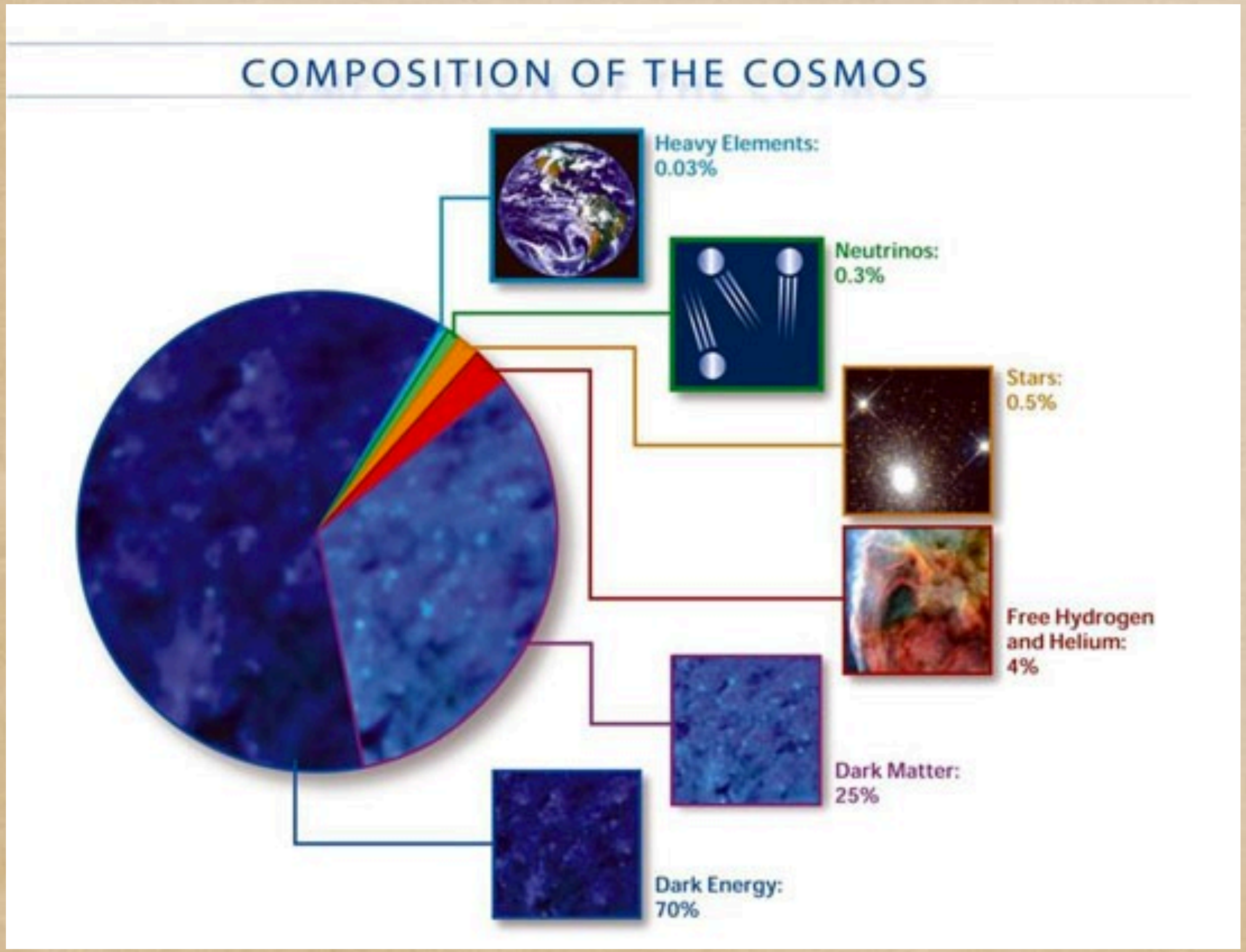


Real universe

Astro 101

Dark energy is real

Dark energy — acceleration of the cosmic expansion



DE vs MG

Matter

DE: new exotic matter

(quintessence/ K-essence/...)

Geometry

MG: new terms/ DOF

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}$$

Cosmological constant

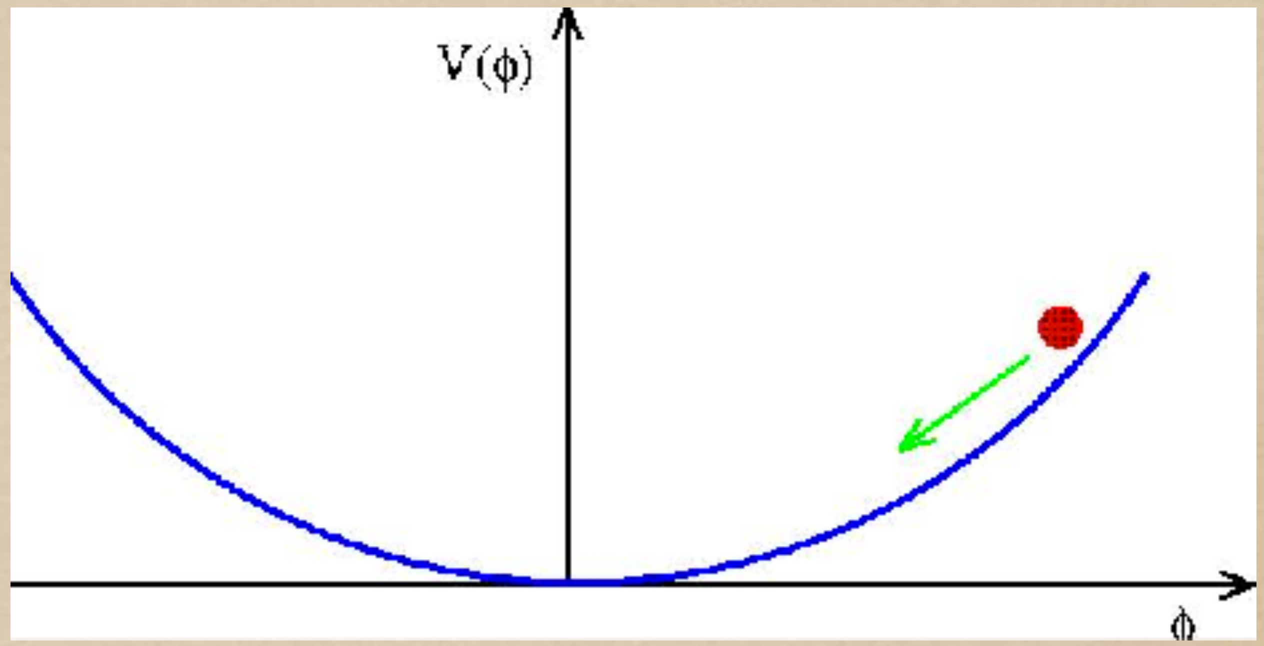
(drives acceleration but fine-tuned)

(DE: zero due to symmetry/dynamics)

If your life doesn't make sense, buy some Dark Energy and balance your equations. We won't promise that the rest of the world will make any more sense, but at least you'll have beer.

Cosmological scalars

Mass m, coupling β



$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + 8\pi\beta G\rho = 0$$

Friction Driving Forcing (from MG)

'Physics: the study of the harmonic oscillator'

Cosmological scalars

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + 8\pi\beta G\rho = 0$$

Friction Driving Forcing (from MG)

Want:

$$m \sim H_0$$ (under-damped, field rolls today)

$$\beta \geq 1$$ (MG non-negligible)

2) Why we need to screen

Yukawa forces

$$\nabla^2\phi + m^2\phi = 8\pi\beta G\rho$$

Strength of fifth-force $\alpha = 2\beta^2$

$$V(r) = \frac{GM}{r}\left[1 + 2\beta^2 e^{-mr}\right]$$

Structure Formation

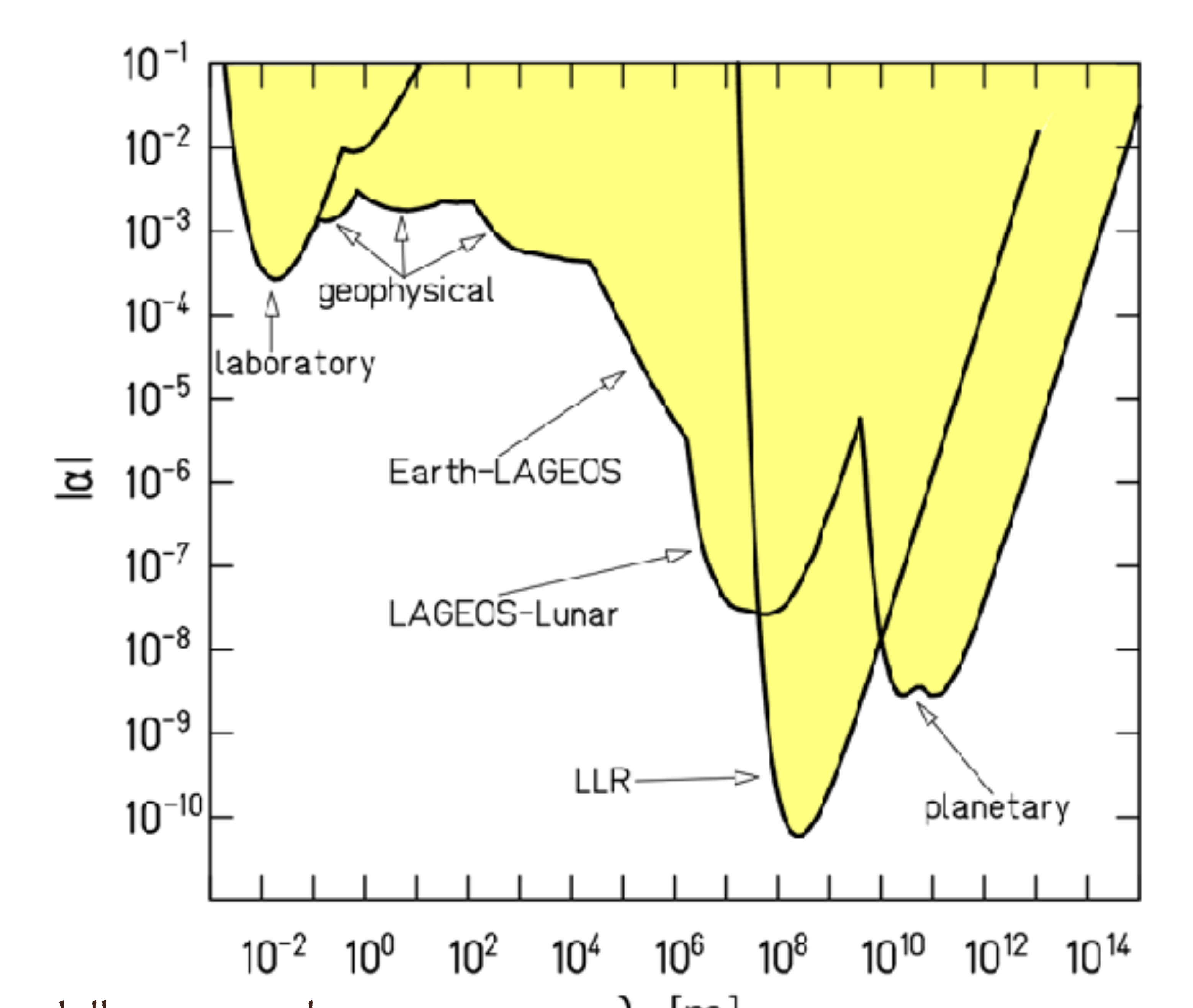
Force range $\lambda = \frac{\hbar c}{m}$

Cosmology: $\text{Mpc} \leq \lambda \leq 10^5\ \text{Mpc}$

Expansion

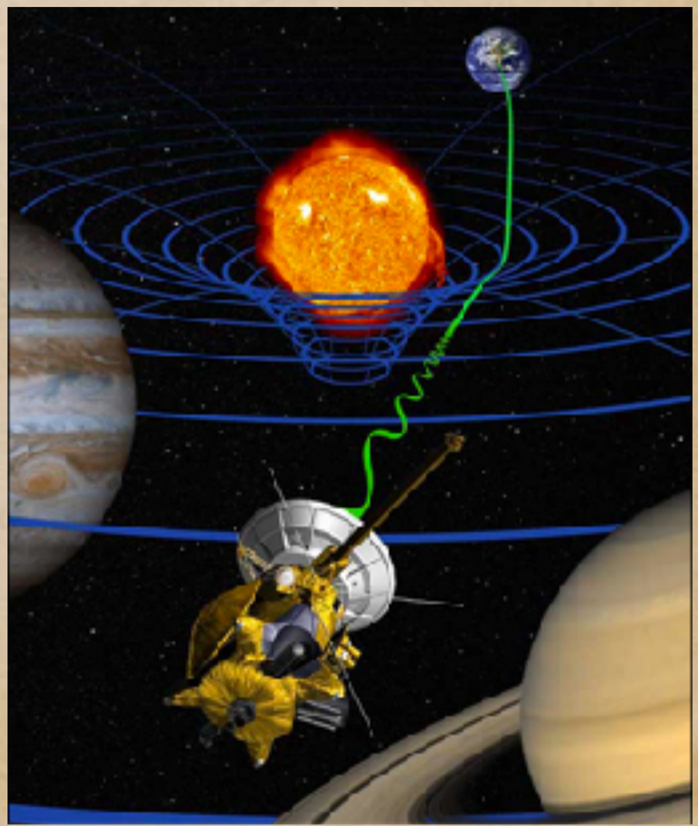
Yukawa forces

Cosmology



Adelberger et al '09

Ruled out by Shapiro time-delay (PPN in general)

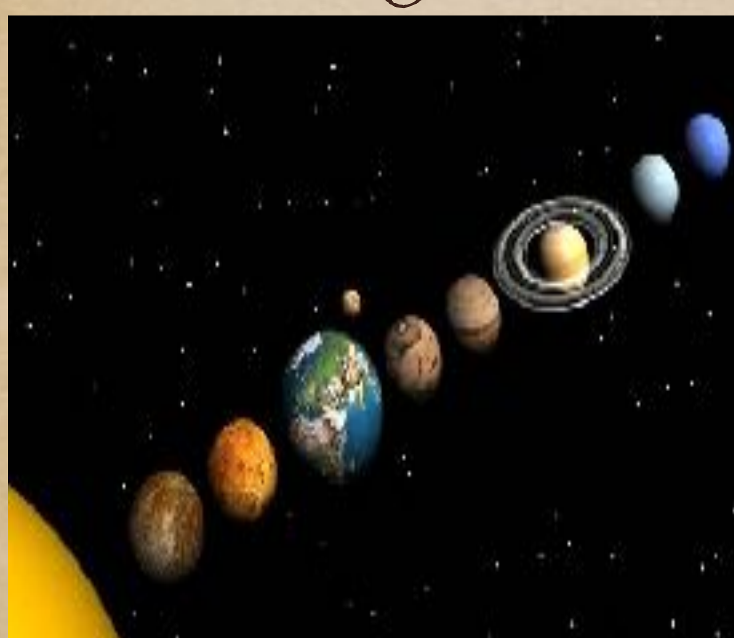


This is why we need to screen

satisfy fifth-forces

drive acceleration

solar system



screened

astrophysics

partially screened

cosmology

unscreened

3) Principles of screening

Where is the problem?

Equation is linear:

$$\Box\phi + m^2\phi = 8\pi\beta G\rho$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + 8\pi\beta G\rho = 0$$

cosmological oscillator

$$\nabla^2\phi + m^2\phi = 8\pi\beta G\rho$$

solar system Yukawa

need non-linearities - scale dependence!

Where is the problem?

$$\nabla^2\phi + m^2\phi = 8\pi\beta G\rho$$

source term

cosmology - needs to be big

fifth-force - needs to be small

Poisson part

gives inverse-square law

mass term

cosmology - needs to be small

fifth-force needs to be big

Suggests solution

Non-linear potential
(environment-dependent mass)
chameleon mechanism

$$\nabla^2\phi + F(\partial\phi, \partial^2\phi, \ldots) + m^2\phi + V(\phi) = 8\pi\beta(\phi)G\rho$$

change kinetic term
(non-inverse square)
Vainshtein mechanism

non-linear coupling
(environment-dependent source)
symmetron mechanism

1) Chameleon mechanism

Action

$$S = M_{\rm pl}^2 \int {\rm d}^4x \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_m[A^2(\phi)g_{\mu\nu}]$$

scalar-tensor theory with coupling $A(\phi) = e^{\beta\phi}$

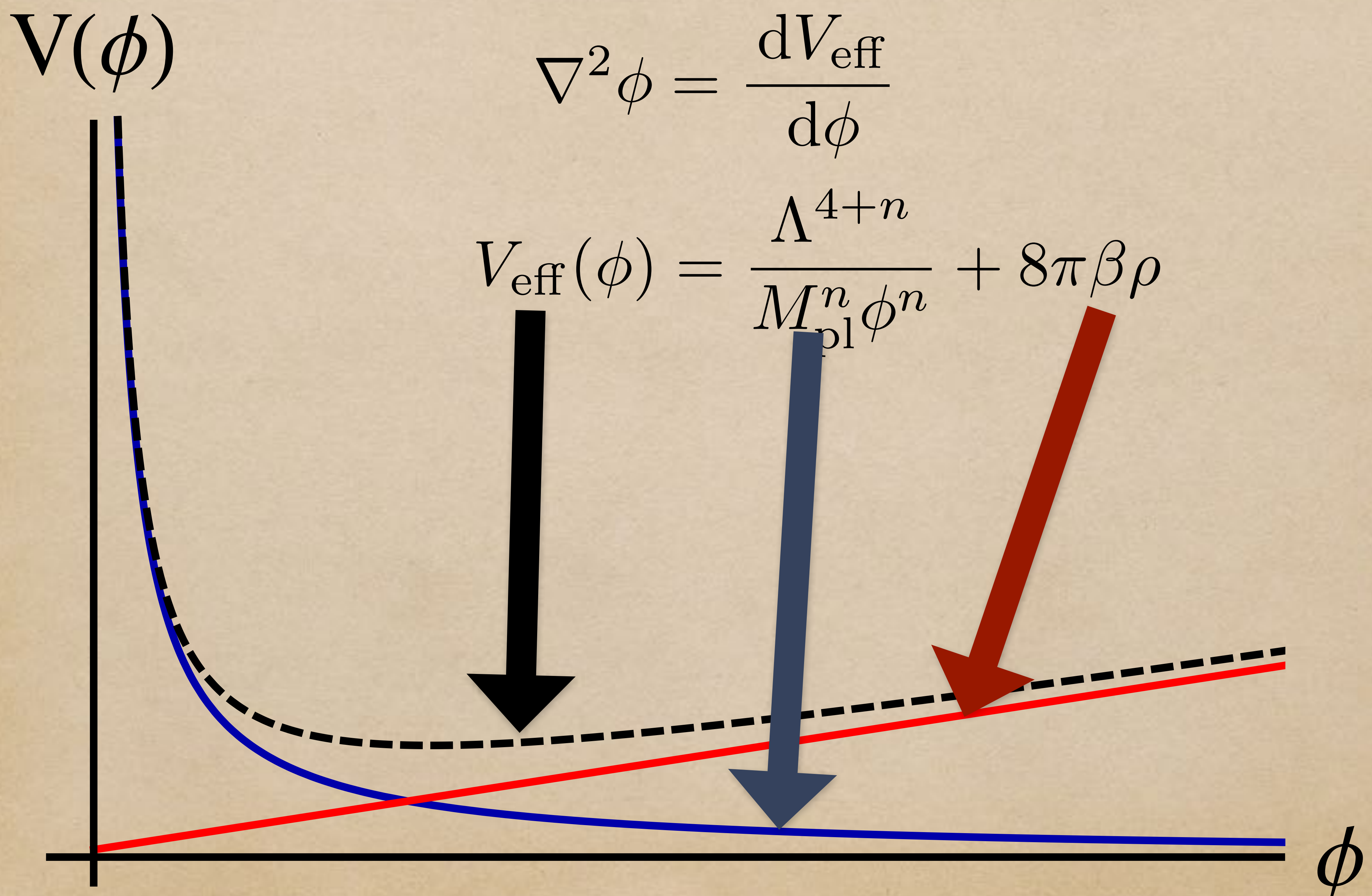
fifth-force: $F_5 = \beta\nabla\phi$

choose $V(\phi) = \dfrac{\Lambda^{4+n}}{M_{\rm pl}^n\phi^n}$

Effective potential

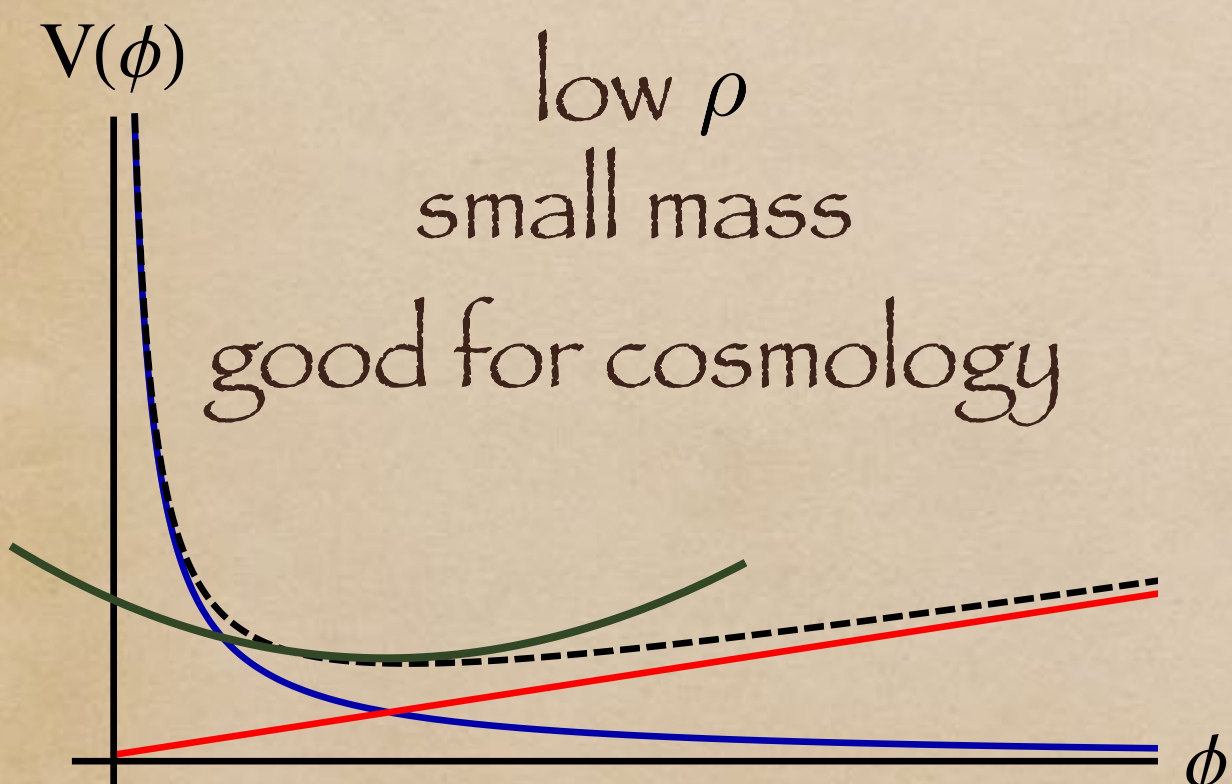
$$\nabla^2\phi = \frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}\phi}$$

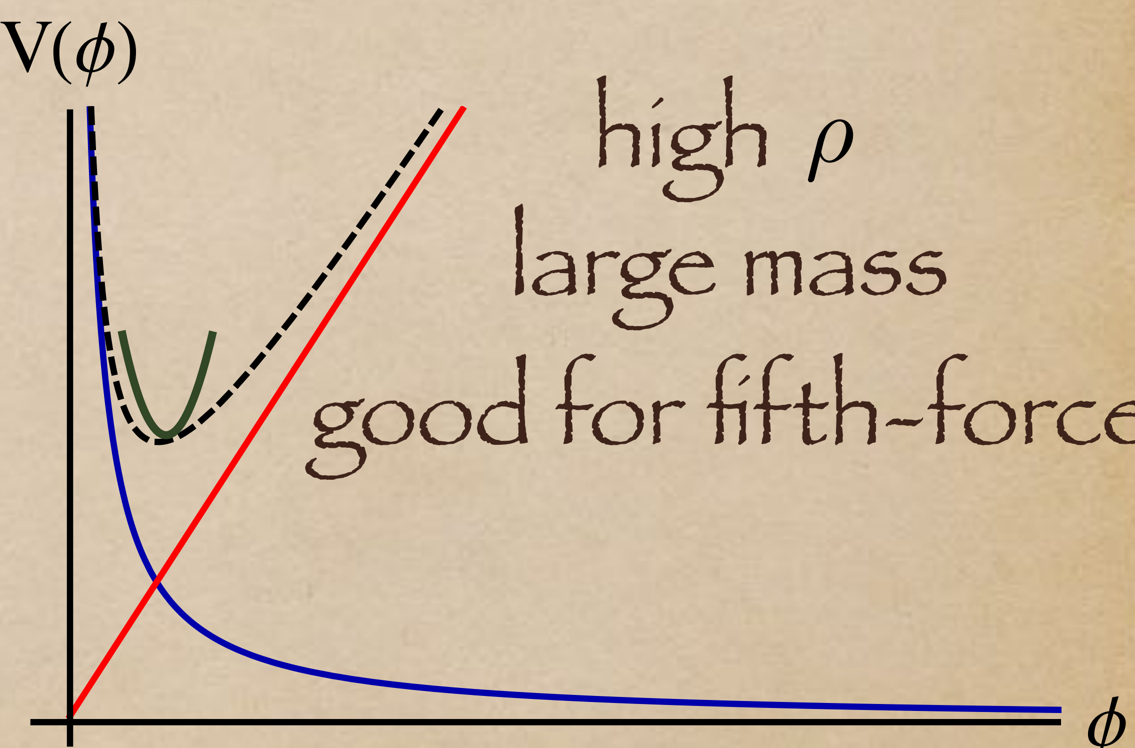
$$V_{\mathrm{eff}}(\phi) = \frac{\Lambda^{4+n}}{M_{\mathrm{pl}}^n\phi^n} + 8\pi\beta\rho$$



Environment-dependent mass

$$m_{\text{eff}}^2 = V_{\text{eff}}''(\phi) \propto \rho^{\frac{n+2}{n+1}}$$

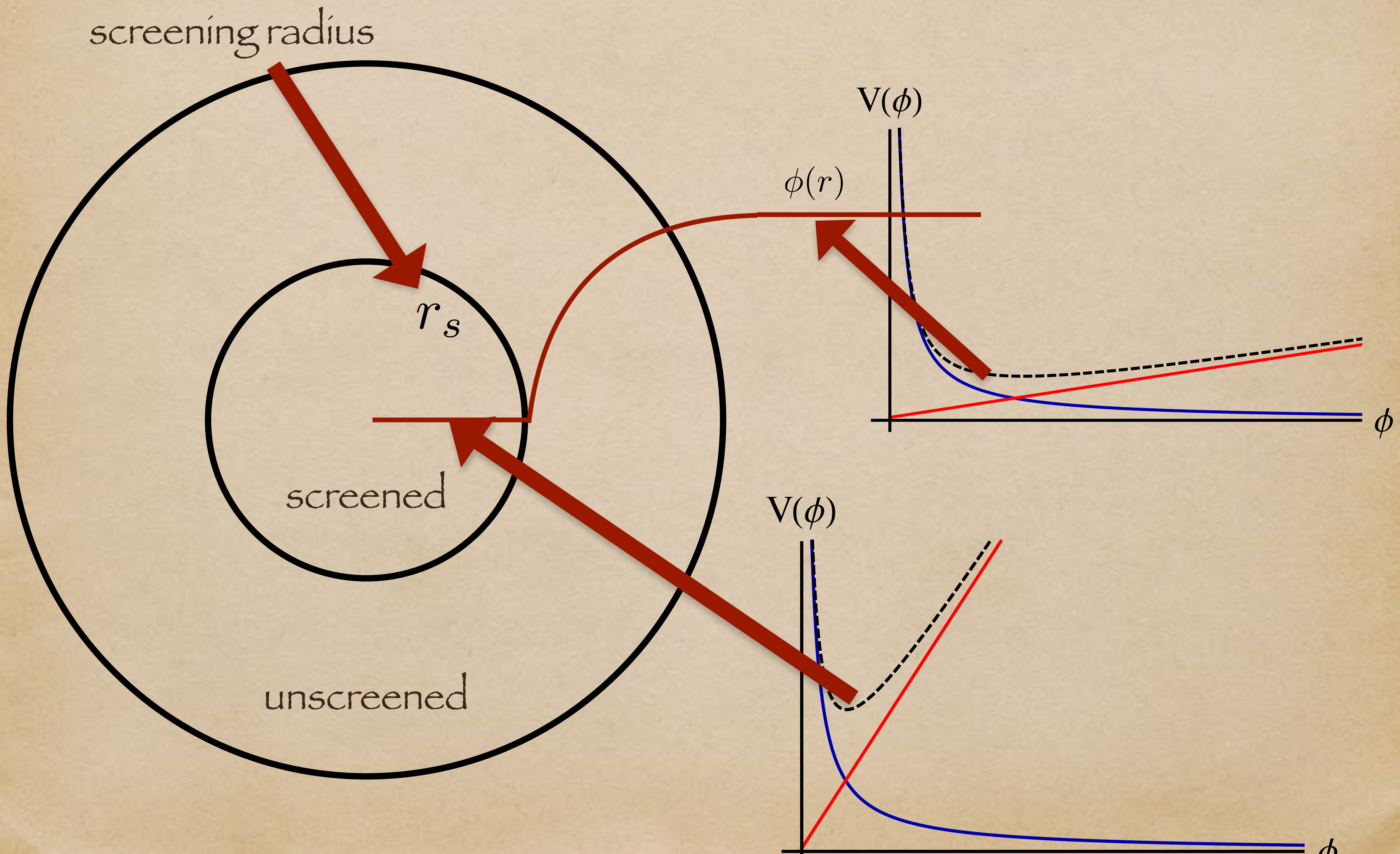




Screening: the thin-shell effect

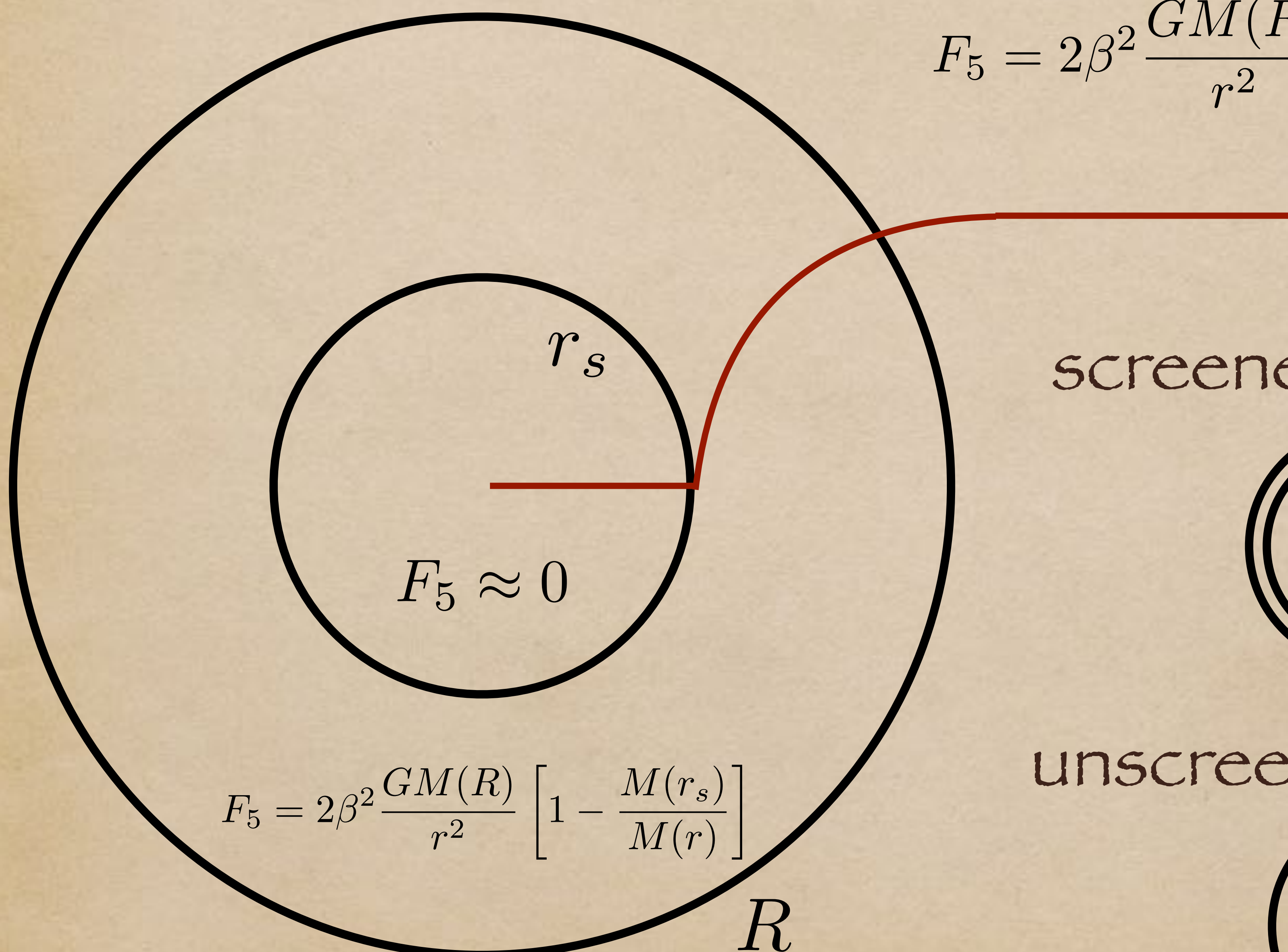
low density

Screening: the thin-shell effect

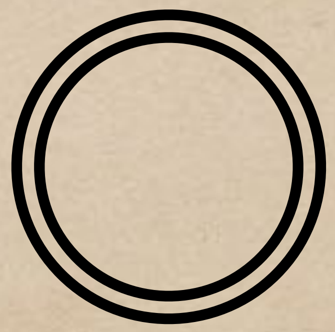


Screening: the thin-shell effect

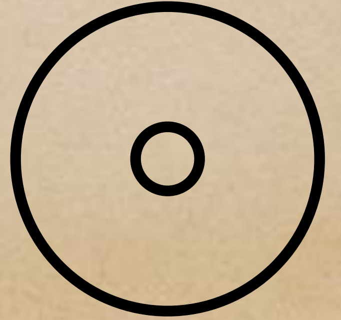
$$F_5 = 2\beta^2 \frac{GM(R)}{r^2}\left[1 - \frac{M(r_s)}{M(r)}\right] e^{-m_{\rm eff} r}$$



screened: $r_s \approx R$



unscreened: $r_s \ll R$



f(R) and chameleons

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R} + f(\tilde{R})\right] + S_m[\tilde{g}]$$

Introduce Lagrange multiplier:

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R} + f(\chi) + \frac{\mathrm{d}f}{\mathrm{d}\chi}\left(\tilde{R} - \chi\right)\right] + S_m[\tilde{g}]$$

Action is equivalent because $\chi = \tilde{R}$

f(R) and chameleons

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R} + f(\chi) + \frac{\mathrm{d}f}{\mathrm{d}\chi}\left(\tilde{R}-\chi\right)\right] + S_m[\tilde{g}]$$

Weyl rescaling: $\tilde{g}_{\mu\nu} = A^2(\chi)g_{\mu\nu}$ $\qquad A^2(\chi) = 1 + \frac{\mathrm{d}f}{\mathrm{d}\chi}$

Canonically normalize: $\phi = -\sqrt{\frac{3}{2}}\ln\left[1+f'(\chi)\right]$

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_m[e^{\sqrt{\frac{2}{3}}\phi}g_{\mu\nu}]$$

$$V(\phi) = \frac{\phi f'(\phi) - f(\phi)}{(1+M_{\rm pl}^{-2}f'(\phi))^2}$$

f(R) and chameleons

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-\tilde{g}}\left[R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_m[e^{\sqrt{\frac{2}{3}}\phi}g_{\mu\nu}]$$

chameleon with $\beta = \frac{1}{\sqrt{6}}$

Have to choose f(R) e.g. Hu & Sawicki:

chameleon with

$$f(R) = -a\frac{\mu^2}{1+(R/\mu^2)^{-b}}$$

$$n = -\frac{b}{b+1}$$

$$V(\phi) = (1+b)M_{\rm pl}^2\mu^2\left(\sqrt{\frac{3}{2}}\frac{ab}{\phi}\right)^{-\frac{b}{1+b}}$$

$$-1 < n < -1/2$$

5) Symmetron mechanism

Action

$$S = M_{\mathrm{pl}}^2 \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right] + S_m[A^2(\phi) g_{\mu\nu}]$$

scalar-tensor theory with coupling $A(\phi) = 1 + \alpha \frac{\phi^2}{2}$

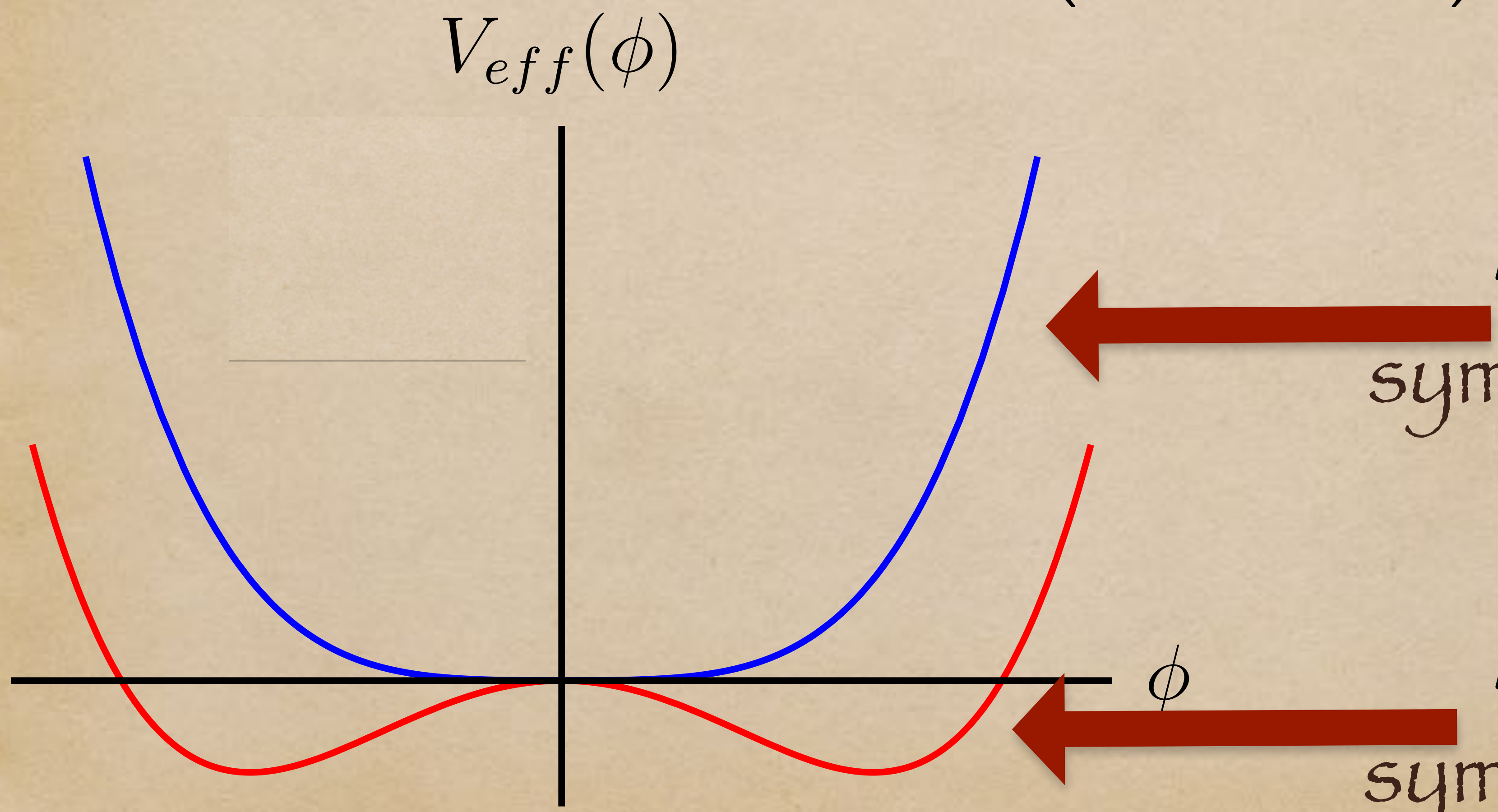
fifth-force: $F_5 = \beta(\phi) \nabla \phi$ $\beta(\phi) = \alpha \phi$

choose $V(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{\lambda}{4} \phi^4$

Effective potential

$$\nabla^2\phi = \frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}\phi}$$

$$V_{\mathrm{eff}}(\phi) = -\frac{1}{2}\mu^2\phi^2\left(1-\alpha\frac{\rho}{M_{\mathrm{pl}}^4}\right)+\frac{\lambda}{4}\phi^4$$



$\rho > M_{\mathrm{pl}}^4/\alpha$

symmetry restored

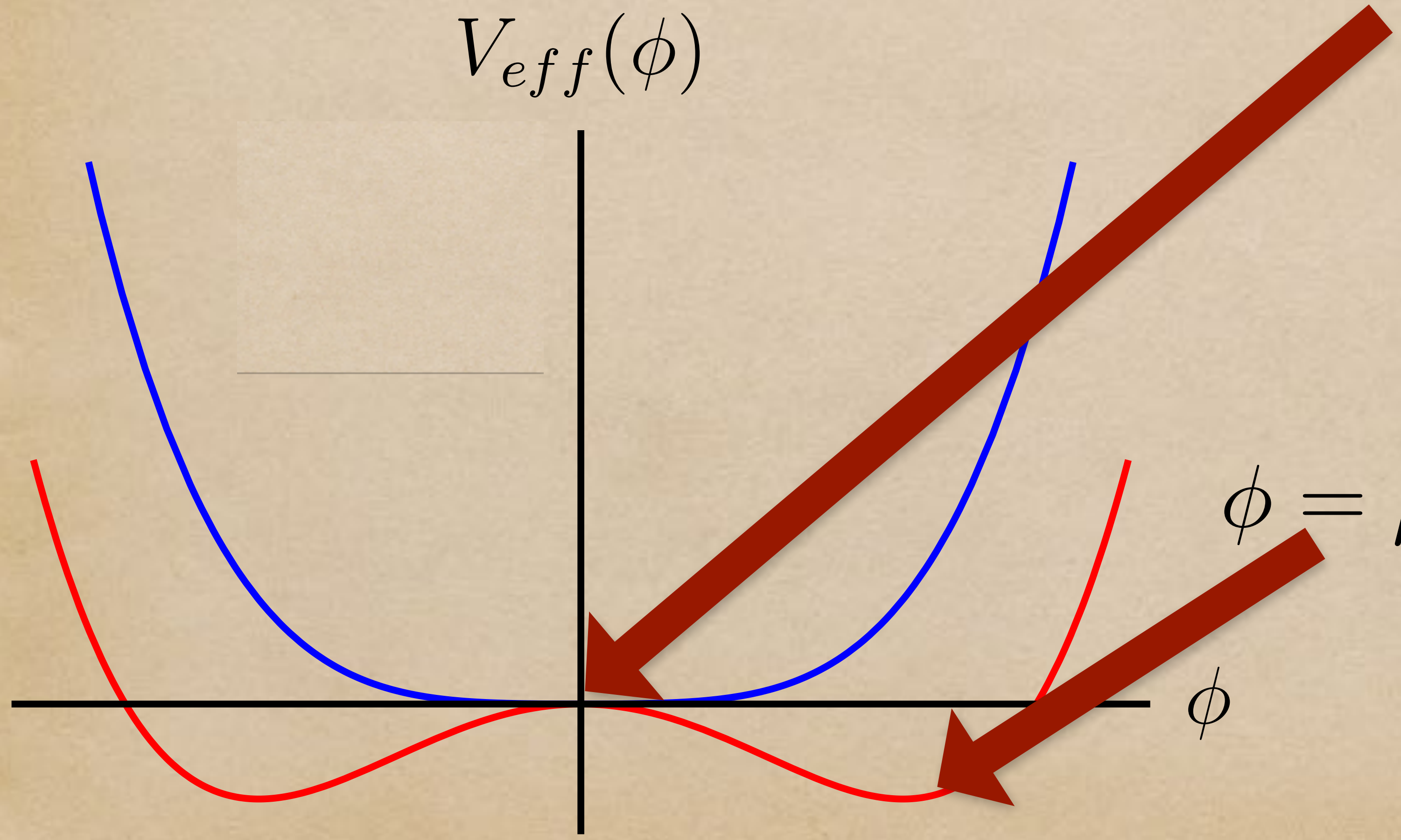
$\rho < M_{\mathrm{pl}}^4/\alpha$

symmetry broken

Effective potential

$$F_5 = \beta(\phi)\nabla\phi$$

$$\beta(\phi) = \alpha\phi$$



$\phi = 0 \Rightarrow \beta = 0$

screened

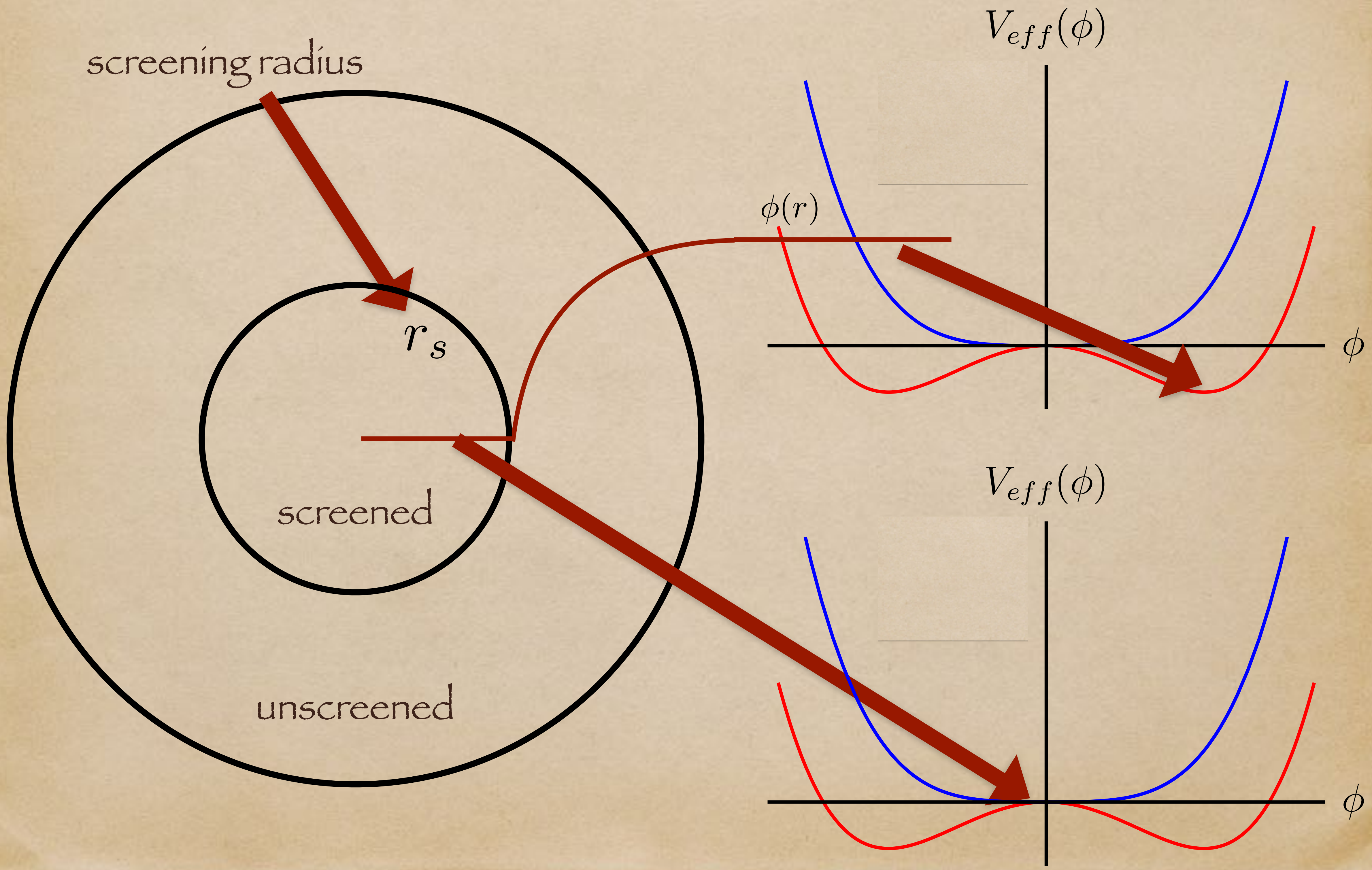
$\phi = \mu/\sqrt{\lambda} \Rightarrow \beta = \alpha\mu/\sqrt{\lambda}$

unscreened

Screening: the thin-shell effect

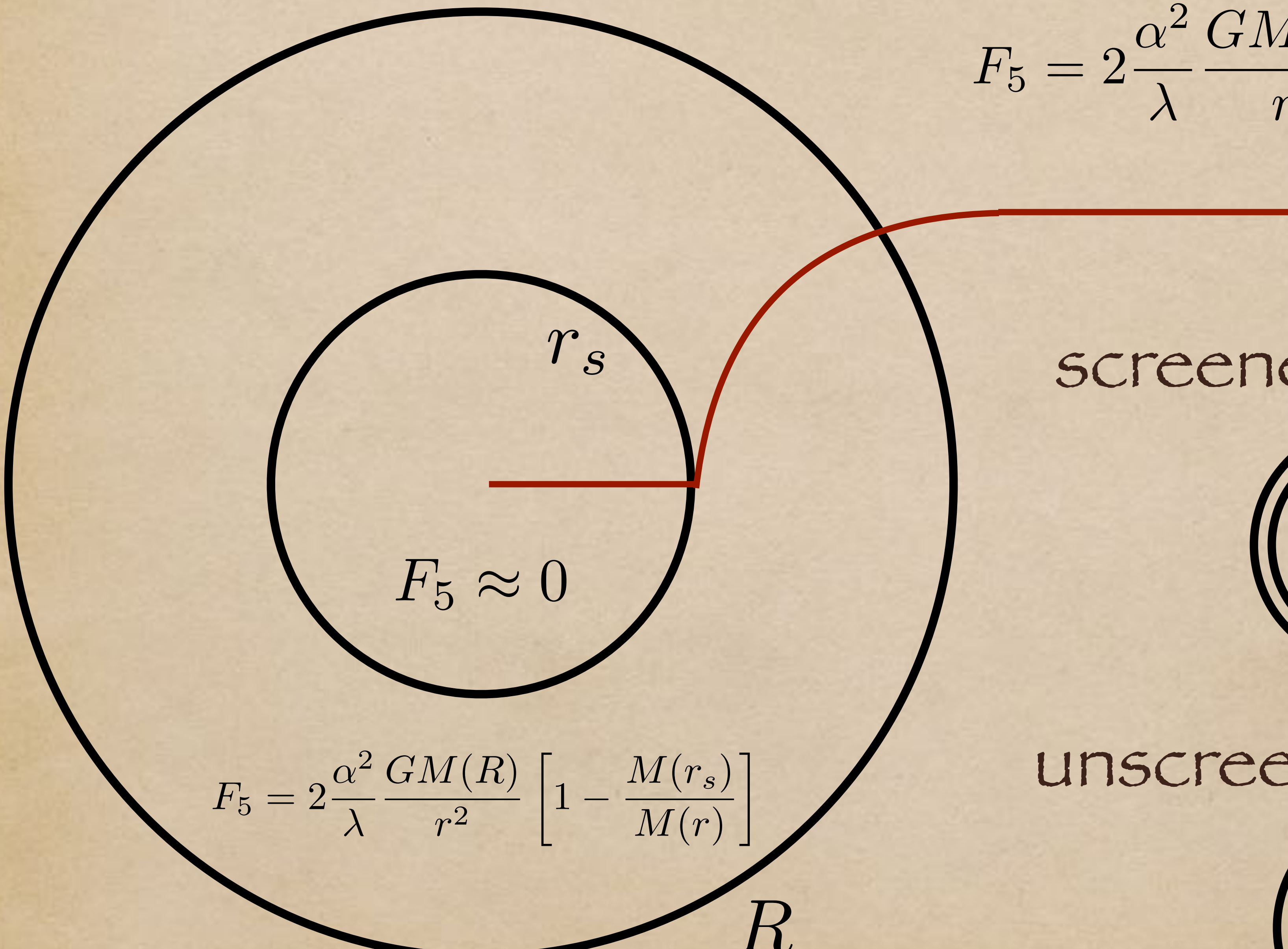
low density

Screening: the thin-shell effect

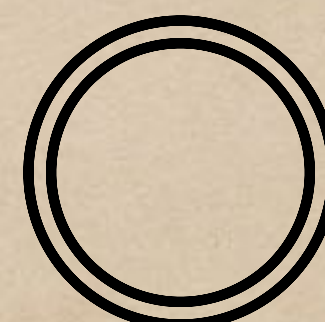


Screening: the thin-shell effect

$$F_5 = 2\frac{\alpha^2}{\lambda}\frac{GM(R)}{r^2}\left[1 - \frac{M(r_s)}{M(R)}\right]e^{-\mu r}$$



screened: $r_s \approx R$

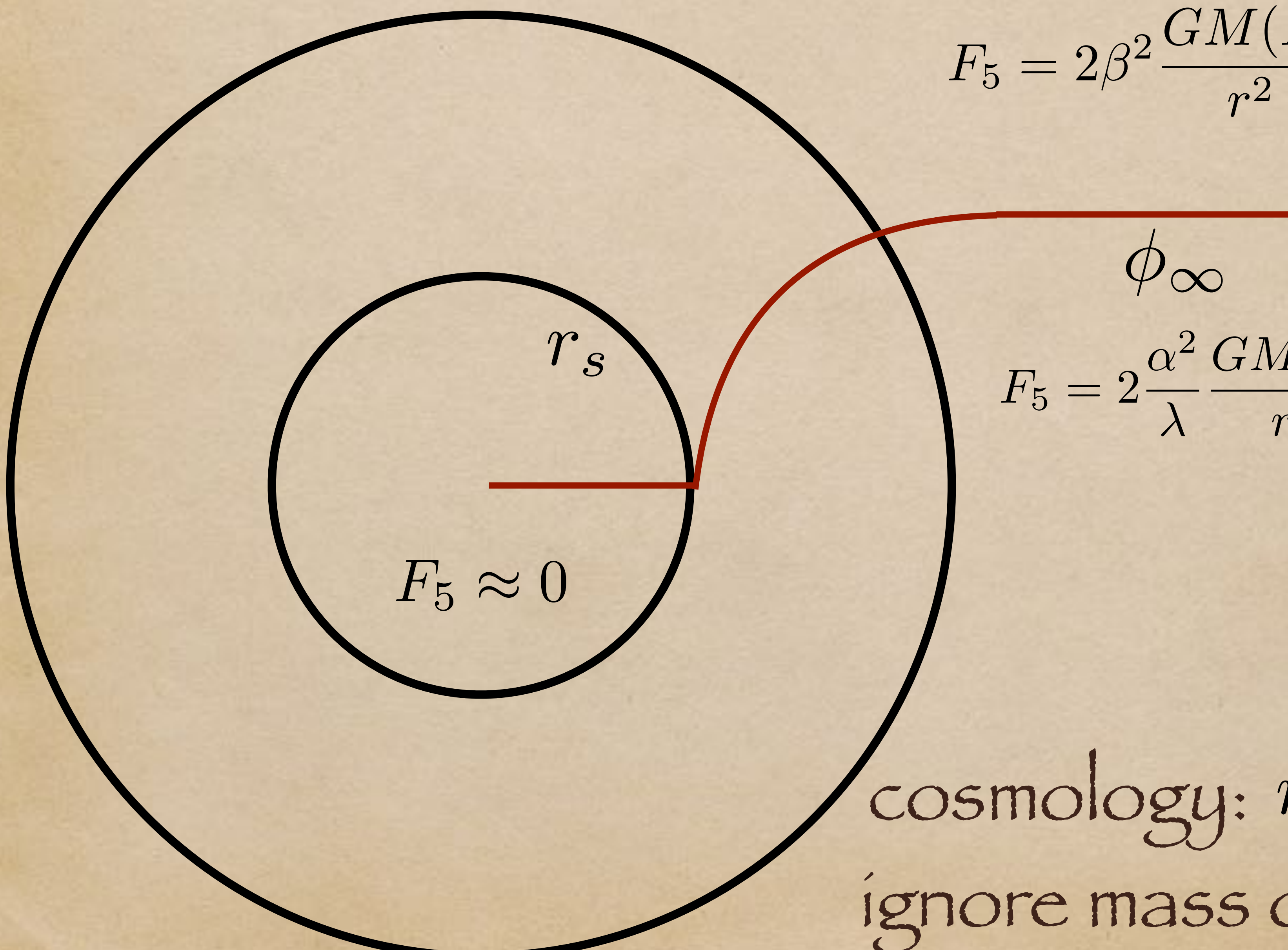


unscreened: $r_s \ll R$

6) Unified description of screening

Astrophysical screening

I.e. what everyone in this room cares about



$$F_5 = 2\beta^2 \frac{GM(R)}{r^2}\left[1 - \frac{M(r_s)}{M(r)}\right] e^{-m_{\rm eff}\, r}$$

(chameleon)

$$F_5 = 2\frac{\alpha^2}{\lambda}\frac{GM(R)}{r^2}\left[1 - \frac{M(r_s)}{M(R)}\right] e^{-\mu r}$$

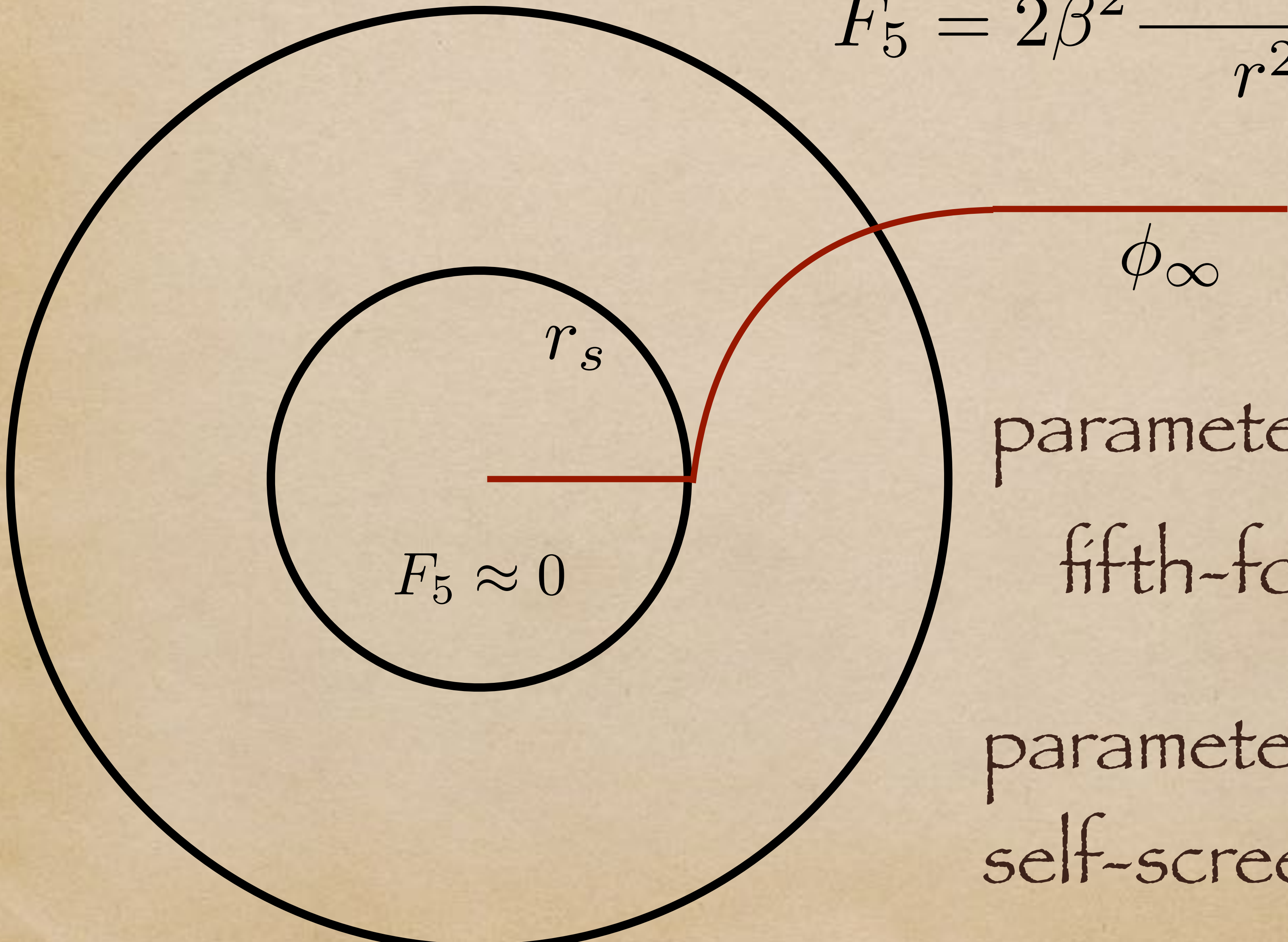
(symmetron)

cosmology: $m_{\rm eff} \sim \mu \sim {\rm Gpc}^{-1}$

ignore mass on small scales

Astrophysical screening

$$F_5 = 2\beta^2 \frac{GM(R)}{r^2}\left[1 - \frac{M(r_s)}{M(R)}\right]$$



parameter 1: β

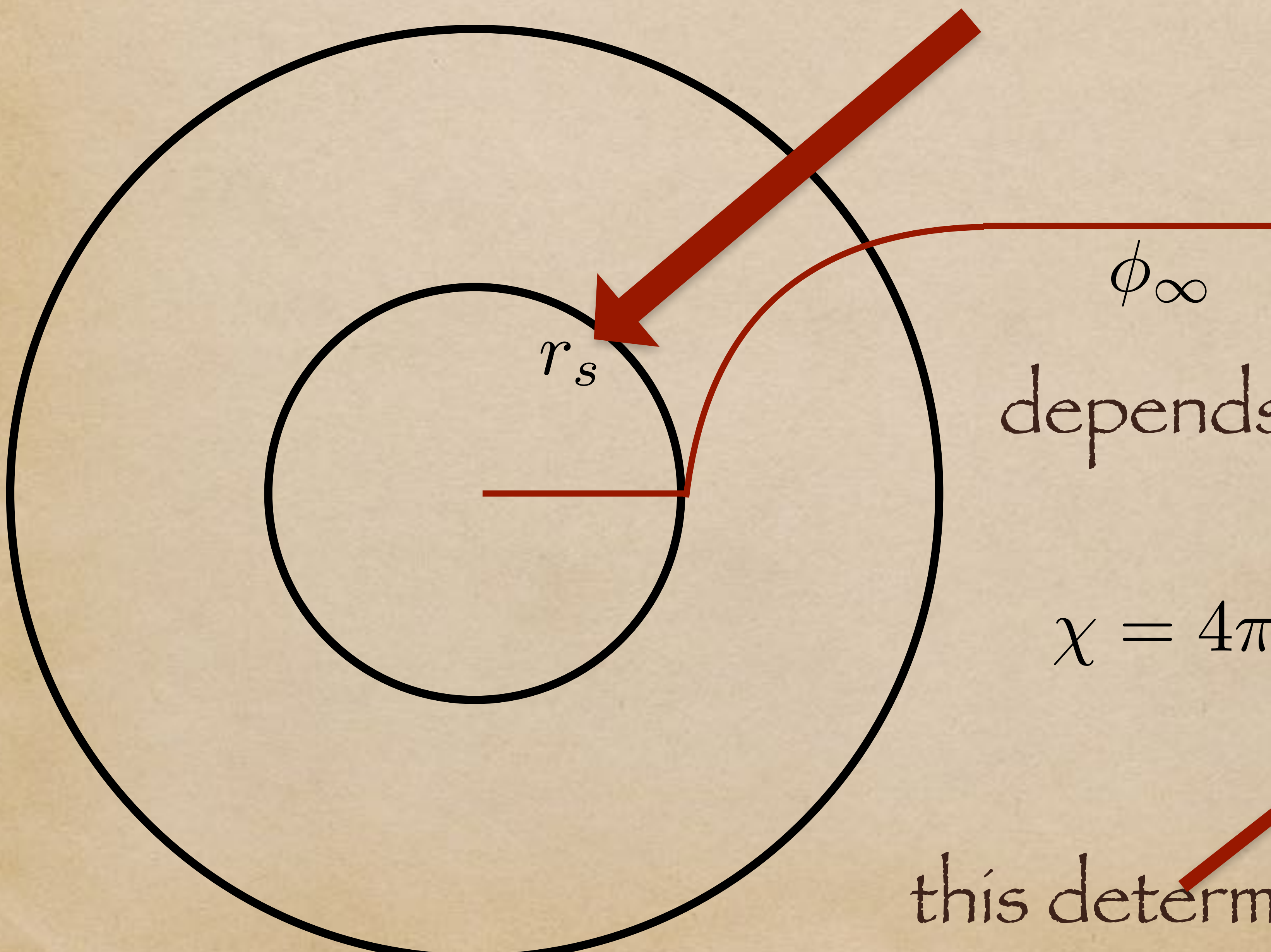
fifth-force strength

parameter 2: χ

self-screening parameter

Self-screening parameter

screening radius depends on object



$$\chi \equiv \frac{\phi_\infty}{2\beta}$$

depends on composition

$$\chi = 4\pi G \int_{r_s}^{R} r\rho(r)\mathrm{d}r$$

this determines r_s

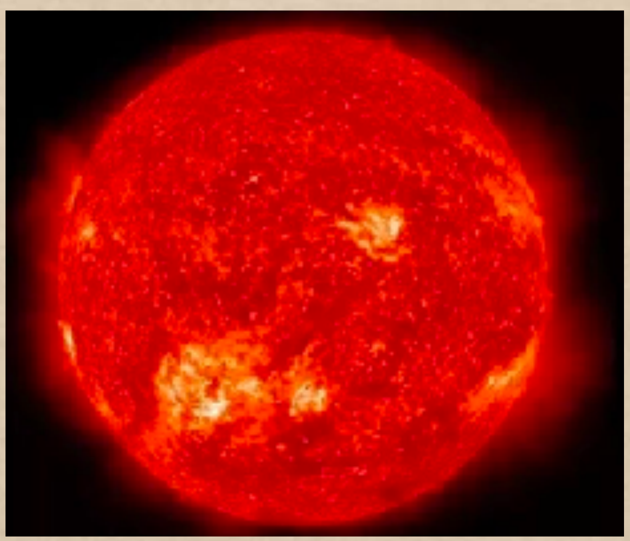
Self-screening parameter

Screened when: $\chi_0 < \Phi_{\rm N} = \frac{GM}{R}$

This tells us where to look:

main-sequence

$\Phi_N \sim 10^{-6}$



red giant

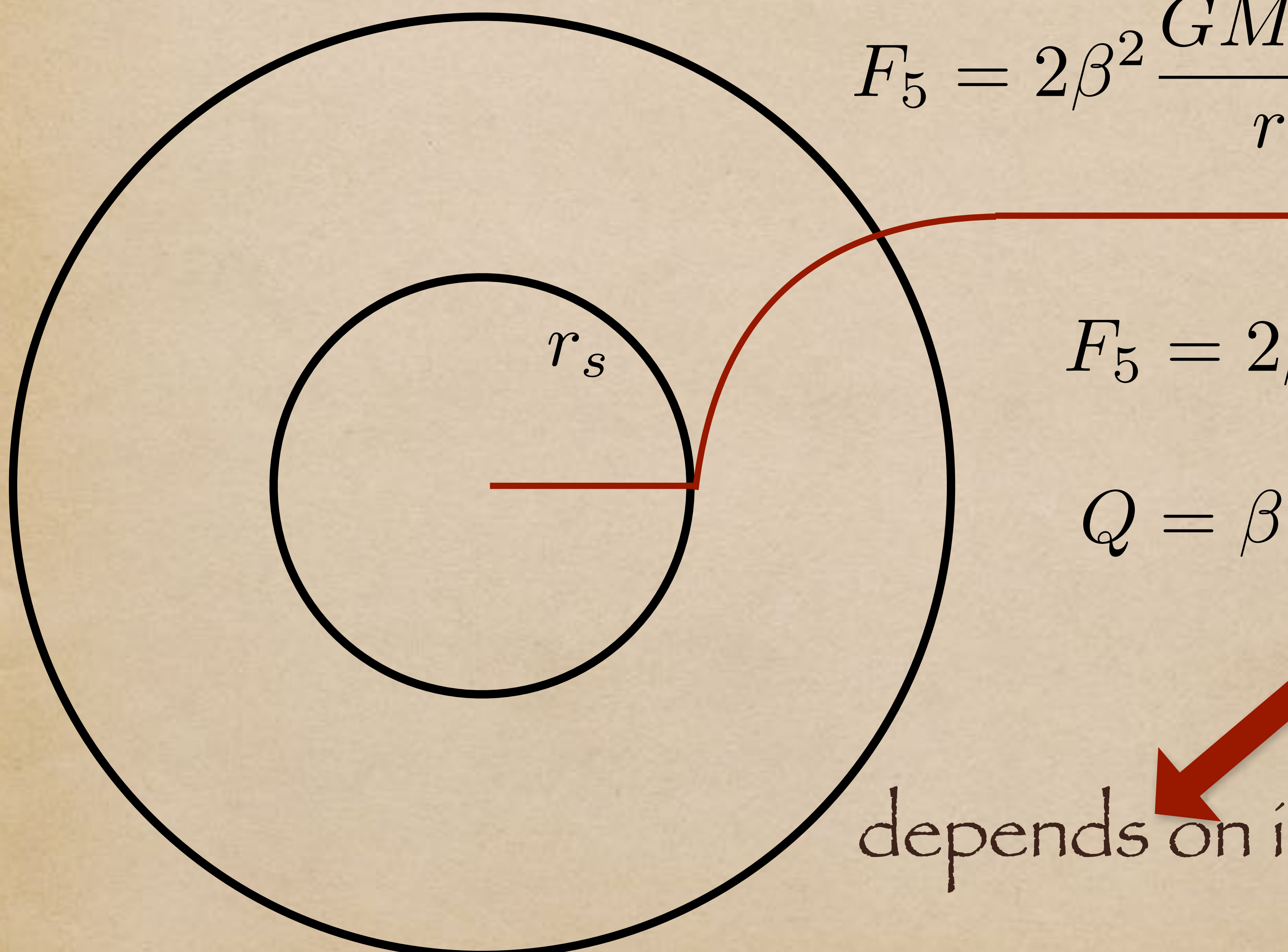
$\Phi_N \sim 10^{-7}$

neutron star

$\Phi_N \sim 10^{-1}$

Weak equivalence principle

$$F_5 = 2\beta^2 \frac{GM(R)}{r^2}\left[1 - \frac{M(r_s)}{M(R)}\right]$$



$$F_5 = 2\beta Q \frac{GM}{r^2}$$

$$Q = \beta\left[1 - \frac{M(r_s)}{M}\right]$$

depends on internal structure

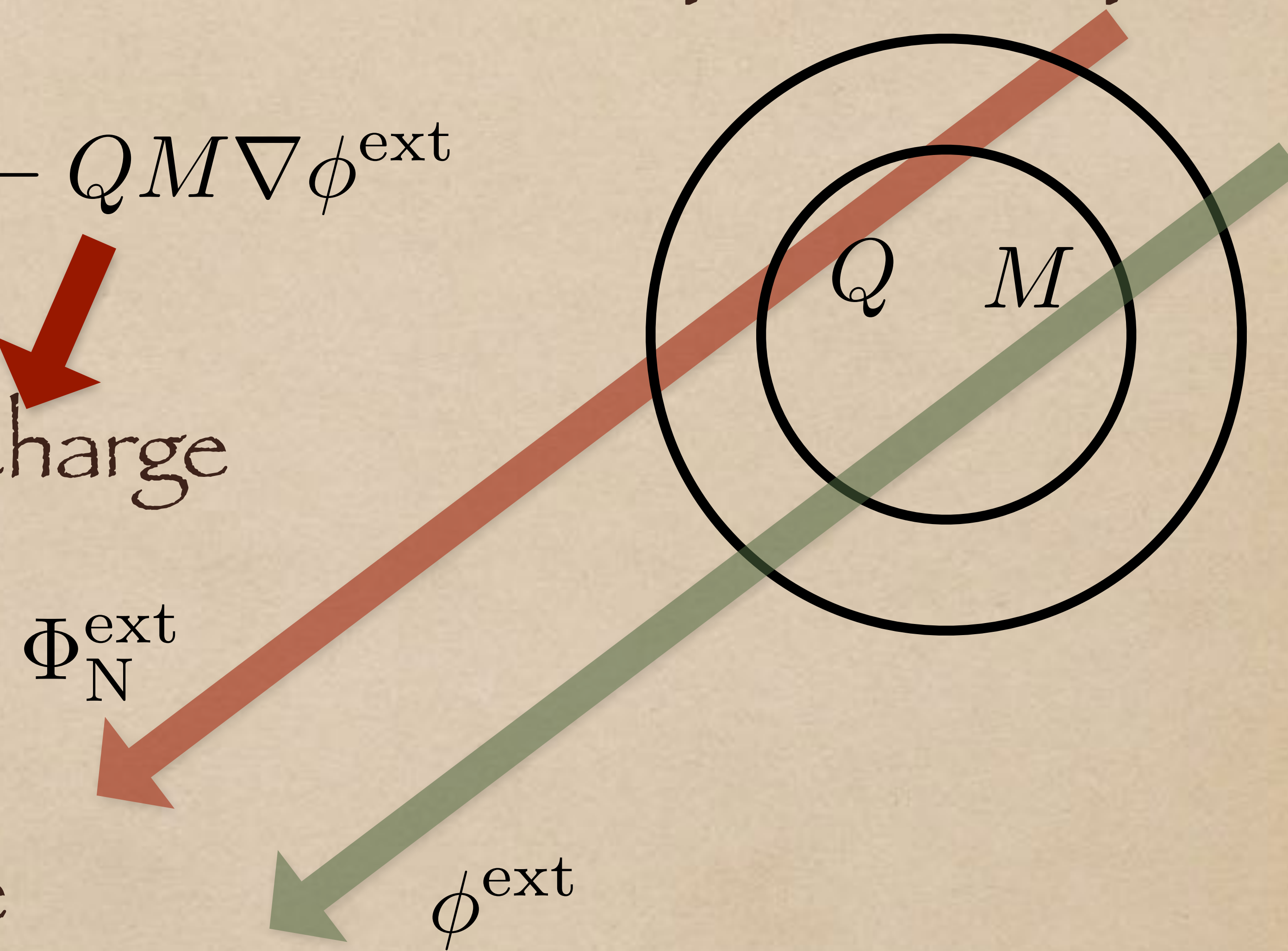
Violation of weak equivalence principle!

Weak equivalence principle

$$M\ddot{\vec{r}} = -M\nabla\Phi_{\rm N}^{\rm ext} - QM\nabla\phi^{\rm ext}$$

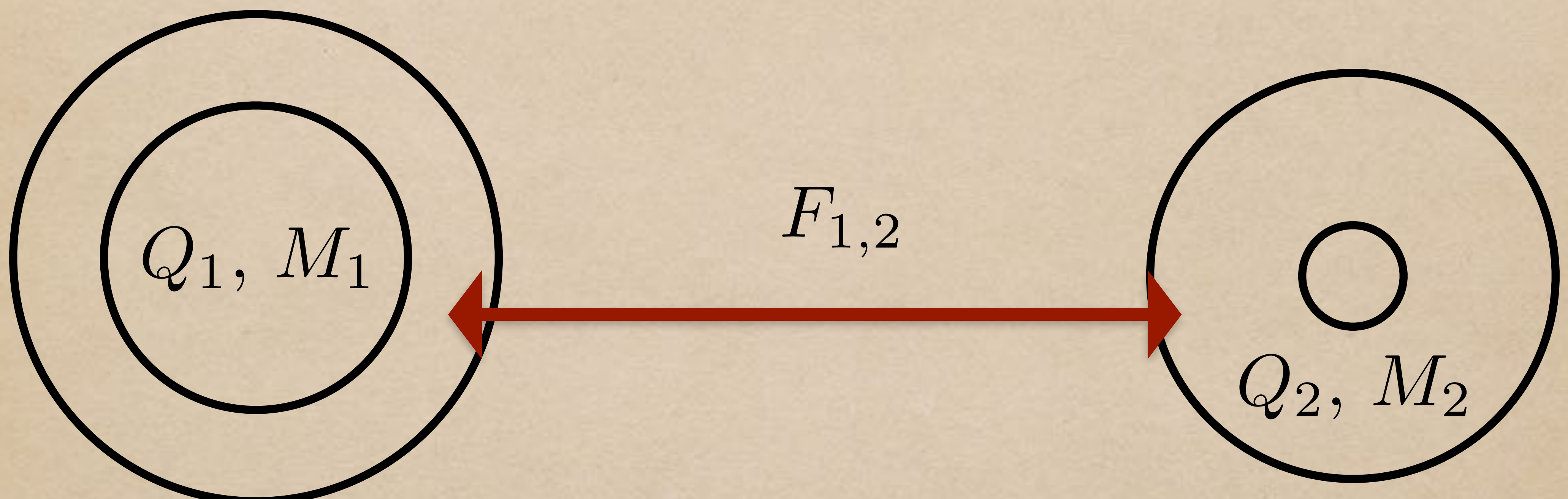
scalar charge

gravitational charge



WEP violated if $Q \neq 1,\, 0$

Force between two objects



$$F_{1,2} = \frac{GM_1M_2}{r^2}\left(1 + 2Q_1Q_2\right)$$

PPN

Normalization of G fixed here

$$g_{00} = -1 + 2\frac{GM}{r} - 2\beta\left(\frac{GM}{r}\right)^2 + \cdots$$

$$g_{ij} = \left(1 + 2\gamma\frac{GM}{R}\right)\delta_{ij}$$

PPN does not include WEP violations!

Example: massless scalar

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,[R - \nabla_\mu\phi\nabla^\mu\phi] + S_m[e^{2\beta\phi}g_{\mu\nu}]$$

Do PPN for the Sun:

$$\nabla^2\phi = 8\pi\beta G\rho \Rightarrow \phi = 2\beta\frac{GM}{r}$$

$$g_{00} = -1 + 2\frac{GM}{r} \Rightarrow \tilde{g}_{00} = -1 + 2(1+2\beta^2)\frac{GM}{r}$$

Example: massless scalar

$$g_{00} = -1 + 2\frac{GM}{r} - 2\beta\left(\frac{GM}{r}\right)^2 + \cdots$$

$$\tilde{g}_{00} = -1 + 2\frac{GM}{r}$$

$$\tilde{g}_{00} = -1 + 2(1+2\beta^2)\frac{GM}{r}$$

$$\tilde{g}_{ij} = \left(1 + 2\frac{(1-2\beta^2)GM}{r}\right)\delta_{ij}$$

$$\tilde{g}_{ij} = \left(1 + 2\frac{(1-2\beta^2)}{1+2\beta^2}\frac{GM}{r}\right)\delta_{ij}$$

$$G \to \frac{G}{1+2\beta^2}$$

$$\gamma = \frac{1-2\beta^2}{1+2\beta^2}$$

Chameleons: naive

$$\phi = 2\beta Q \frac{GM}{r}$$

$$\tilde{g}_{00} = -1 + 2(1 + 2\beta Q)\frac{GM}{r}$$

$$G \to \frac{G}{1 + 2\beta Q}$$

$$\tilde{g}_{ij} = \left(1 + \frac{1 - 2\beta Q}{1 + 2\beta Q}\frac{GM}{r}\right)\delta_{ij}$$

$$\gamma = \frac{1 - 2\beta Q}{1 + 2\beta Q}$$

What is the value of G?

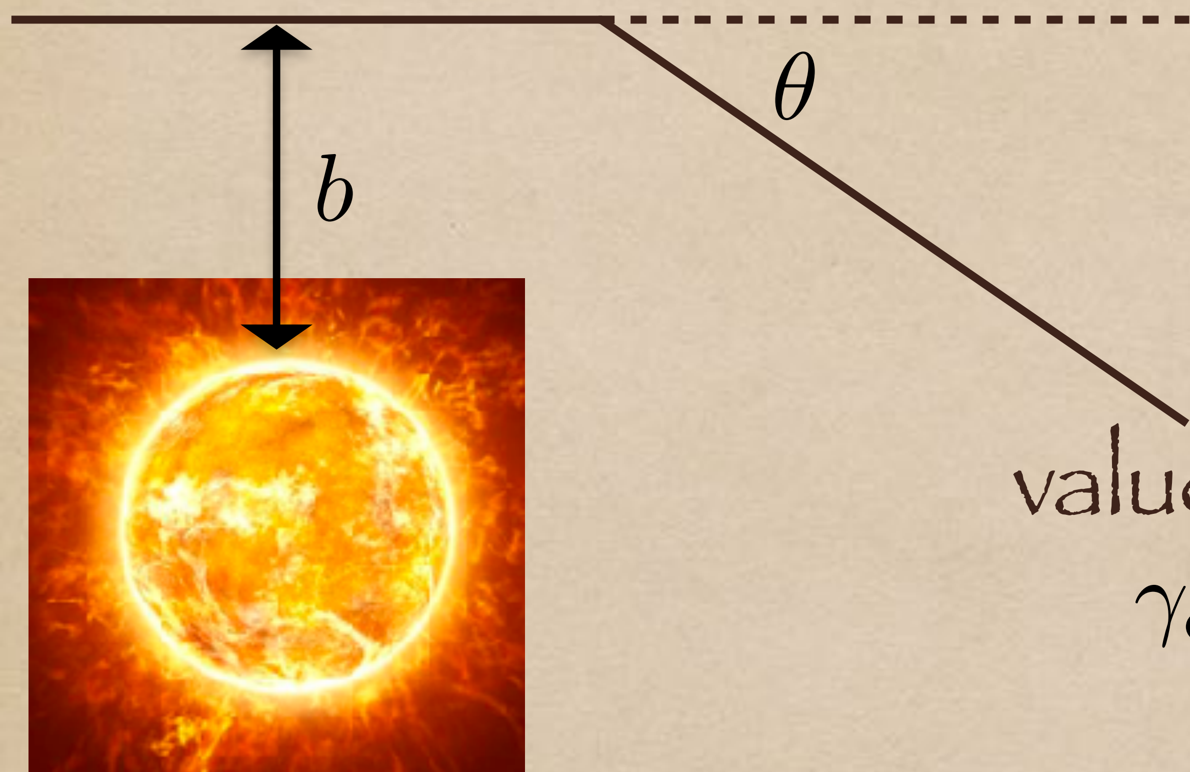
WEP violations are important!

$$F_{\odot,\oplus} = (1 + 2Q_\odot Q_\oplus)\,\frac{G M_\odot M_\oplus}{r^2_{\odot,\oplus}}$$

measured value

$$G_N = \frac{G}{1 + 2Q_\oplus Q_\odot}$$

This effects the measurement



value probed by experiments

$$\gamma_{\text{eff}} = 2(1 + 2Q_{\odot}Q_{\oplus}) - 1$$

$$\theta = \frac{4GM}{b}\left(\frac{1+\gamma}{2}\right) = \frac{4G_{\text{N}}M}{b}(1 + 2Q_{\odot}Q_{\oplus})$$

Constraints weaker than expected!

Screened body: $Q_i \approx \dfrac{GM_i}{R_i}$

$$Q_\odot \sim 10^{-6} \qquad Q_\oplus \sim 10^{-9}$$

$$\gamma = \frac{1-2\beta Q}{1+2\beta Q} \Rightarrow \gamma - 1 \sim 10^{-6}$$

$$\gamma_{\rm eff} = 2(1+Q_\odot Q_\oplus) - 1 \Rightarrow \gamma_{\rm eff} - 1 \sim 10^{-15}$$

Further complications for PPN

- Measuring parameters sensitive to quadruple moment
  (assumes Laplacian structure)
- Solar system dynamics depends on all planets
  (need to account for WEP violations)
- Perturbations from Jupiter important
  (WEP + 3+-body problem)

Black holes

$$S = M_{\rm pl}^2 \int {\rm d}^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_m[A^2(\phi)g_{\mu\nu}]$$

Classic no-hair theorems
Black holes look like GR

May be effects if accretion disks are present

(Davis, Gregory, Jha & Muir'14, Davis, Gregory & Jha '16)

(Neutron stars highly screened)

Outstanding problems

- Can we include WEP in PPN?
- Can we at least have a chameleon-PPN?
- Equivalent calculation for β

7) Vainshtein mechanism and galileons

Vainshtein mechanism

$$\nabla^2\phi + F(\partial\phi, \partial^2\phi, \ldots) + m^2\phi + V(\phi) = 8\pi\beta(\phi)G\rho$$

focus on different kinetic terms

Problem: higher-derivatives introduce ghosts

Example (ignoring dimensions)

$$S = \int \mathrm{d}^4x (\Box\phi)^2$$

Introduce auxiliary field χ

$$S = \int \mathrm{d}^4x (2\chi\Box\phi - \chi^2)$$

Equivalent because EOM: $\chi = \Box\phi$

Field redefinition:

$$\chi = \frac{1}{\sqrt{2}}(\phi_1 - \phi_2)$$

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2)$$

Example (ignoring dimensions)

Integrate by parts:

$$S = \int d^4x \frac{1}{2}\partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2}\partial_\mu \phi_2 \partial^\mu \phi_2 + \cdots$$

wrong-sign kinetic term

ghost!

Galileons to the rescue

Impose Galilean symmetry:

$$\phi \to \phi + b_\mu x^\mu + c$$

Gives four unique terms:

$$\mathcal{L}_2 = \partial_\mu \phi \partial^\mu \phi$$ quadratic

$$\mathcal{L}_3 = \partial_\mu \phi \partial^\mu \phi (\Box \phi)$$ cubic

$$\mathcal{L}_4 = \partial_\mu \phi \partial^\mu \phi [(\Box \phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi]$$ quartic

$$\mathcal{L}_5 = \cdots$$ quintic

Gives second-order equations of motion

e.g. cubic:

$$S = M_{\rm pl}^2 \int \mathrm{d}^4x \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \partial_\mu\phi\partial^\mu\phi\frac{\Box\phi}{\Lambda^2} + \beta\phi T \right]$$

$$\Box\phi + \frac{1}{\Lambda^2}\left[(\Box\phi)^2 - \nabla_\mu\phi\nabla^\mu\phi\right] = -8\pi\alpha G T$$

Vainshtein mechanism

Take non-relativistic limit + spherical symmetry:

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2\phi' + \frac{r\phi'^2}{\Lambda^2}\right] = 8\pi\beta G\rho$$

Poisson galileon

Integrate once:

$$r^2\phi' + \frac{r\phi'^2}{\Lambda^2} = 2\beta\frac{GM}{r^2}$$

Vainshtein mechanism

$$r^2\phi' + \frac{r\phi'^2}{\Lambda^2} = 2\beta\frac{GM}{r^2}$$

Use $F_5 = \beta\phi'; \quad F_N = \dfrac{GM}{r^2}$

$$\frac{F_5}{F_N} + \left(\frac{r_V}{r}\right)^3 \left(\frac{F_5}{F_N}\right)^2 = 2\beta^2$$

$$r_V^3 = \frac{GM}{\Lambda^2}$$ Vainshtein radius

Vainshtein mechanism

$$\frac{F_5}{F_N} + \left(\frac{r_V}{r}\right)^3 \left(\frac{F_5}{F_N}\right)^2 = 2\beta^2$$

$r \ll r_V$

$$\frac{F_5}{F_N} = 2\beta^2 \left(\frac{r}{r_V}\right)^{\frac{3}{2}}$$

$r \gg r_V$

$$\frac{F_5}{F_N} = 2\beta^2$$

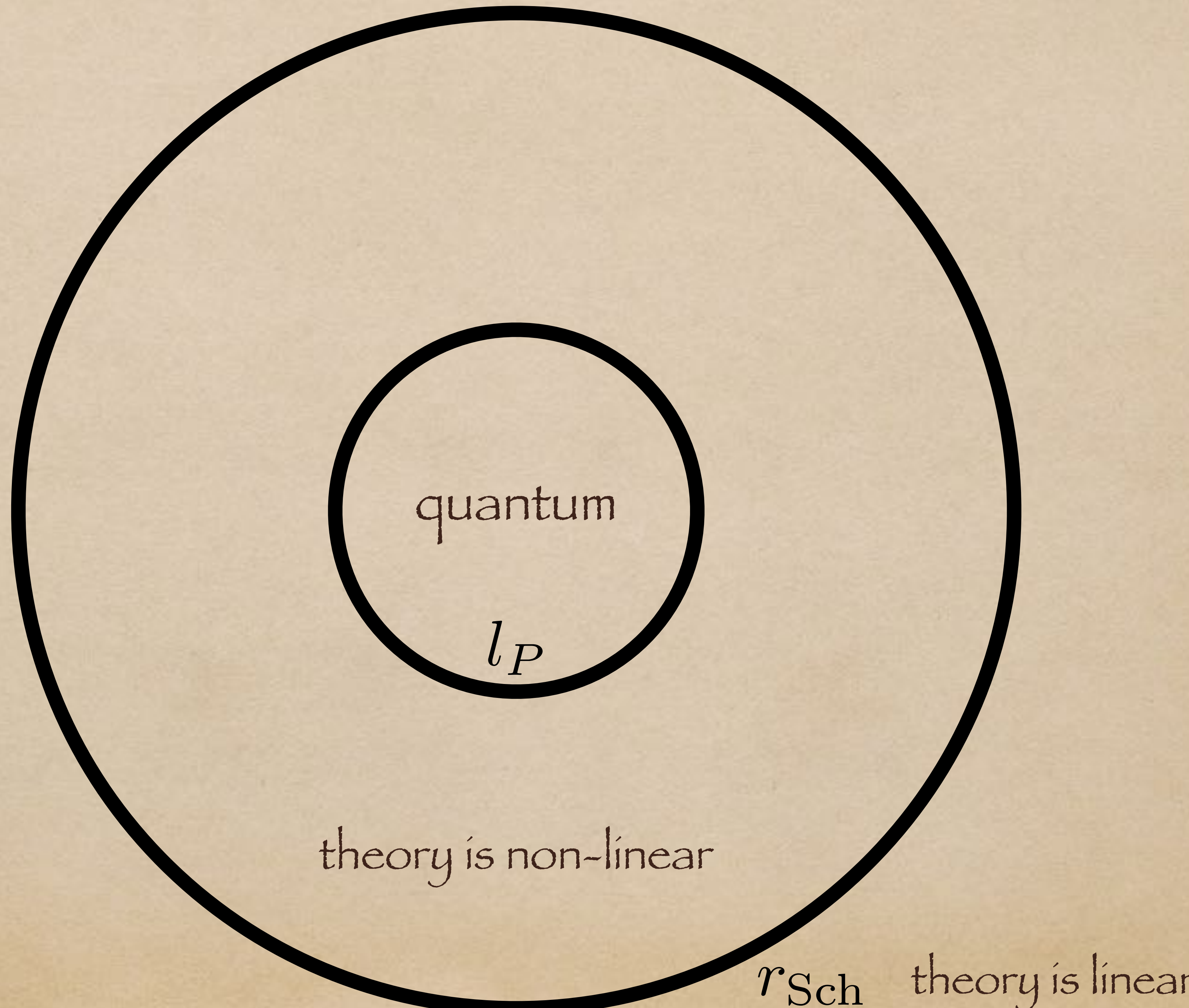
dynamical suppression

order-unity

(good for cosmology)

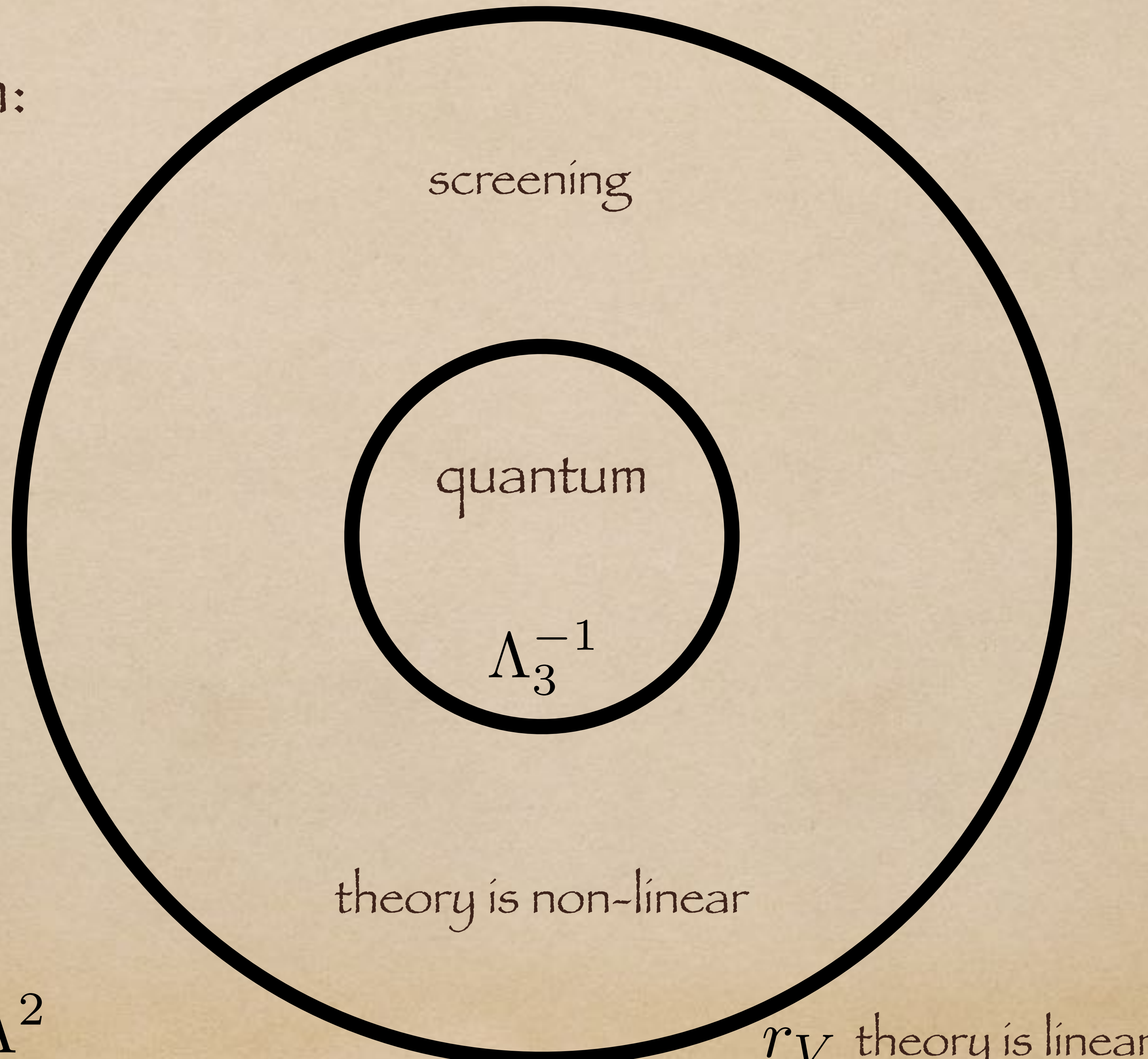
Vainshtein mechanism

GR:



Vainshtein mechanism

Vainshtein:



$\Lambda_3^3 = M_{\rm pl}\Lambda^2$

Weak equivalence principle

WEP is satisfied: $M\ddot{\vec{r}} = -M\nabla\Phi_{\rm N}^{\rm ext} - QM\nabla\phi^{\rm ext}$

$Q = 1$

Harder to find novel probes!

Outstanding problems

- Highly non-linear (hard to compute)
- Only have solutions for symmetric cases
- 2+ body problem difficult, need numerics
- Need to extend PPN (Ryan McManus)
- Perturbation theory often fails
- Equations not quasi-linear — well-posedness?
- Lab and solar system tests hard to model

8) Horndeski and beyond

Motivations

1) Galileons defined on Minkowski space

e.g. quartic $\mathcal{L}_4 = \partial_\mu \phi \partial^\mu \phi [(\Box \phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi]$

Can commute on Minkowski but $[\nabla_\mu, \nabla_\nu] v^\alpha = R^\alpha_{\mu\nu\beta} v^\beta$

Get terms like: $\nabla_\mu R^\mu_{\nu\alpha\beta} \nabla^\mu \phi \nabla^\alpha \nabla^\beta \phi$

third-derivative of the metric = ghost!

Motivations

2) Cosmology - want a framework to model build

Requirements:

- Lorentz-invariant
- Second order EOM (no ghosts)
- Stable (no tachyons, gradient instabilities)
- Free functions
- Screening mechanisms (pass SS tests)
- Falsifiable

Horndeski Action

Want most general action with second-order EOM

quintessence

K-essence

cubic

quartic counter term

$$\frac{\mathcal{L}}{\sqrt{-g}} = K(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$

$$+G_{4,\,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$$

$$-\frac{1}{6}G_{5,\,X}\left[(\Box\phi)^3 - 3\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\Box\phi + 2\nabla^\nu\nabla_\mu\phi + \nabla^\alpha\nabla_\nu\phi\nabla^\mu\nabla_\alpha\phi\right]$$

quartic

quintic

quintic counter term

Features of the Horndeski action

Cosmology: $\phi = \phi(t)$

time-varying G

$$\frac{\mathcal{L}}{\sqrt{-g}} = K(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$
$$+G_{4,\,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$$
$$-\frac{1}{6}G_{5,\,X}\left[(\Box\phi)^3 - 3\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\Box\phi + 2\nabla^\nu\nabla_\mu\phi + \nabla^\alpha\nabla_\nu\phi\nabla^\mu\nabla_\alpha\phi\right]$$

Speed of GWs ≠ c

Vainshtein screening

Beyond Horndeski

Motivation: want healthy Lorentz-invariant theory

$$\{g_{\mu\nu}, \phi\} = 3 \text{ DOF}$$

Horndeski

2nd order EOMs

3 DOF

beyond Horndeski

naively higher-order

extra Hamiltonian constraints

3 DOF

Example: quartic galileon

$$\frac{\mathcal{L}}{\sqrt{-g}} = \left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + 4\nabla^\mu\phi\nabla^\nu\phi\left[\nabla_\mu\nabla_\nu\phi\Box\phi - \nabla_\mu\nabla_\sigma\phi\nabla^\sigma\nabla_\nu\phi\right]$$

beyond Horndeski

(total derivative in flat space)

$$\mathcal{L} = X\left[(\partial_\mu\partial_\nu\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]$$

counter term
kills higher-order

Horndeski

$$\mathcal{L} = \frac{1}{2}X^2 R + X\left[(\nabla_\mu\nabla_\nu\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

beyond beyond Horndeski

DHOST

Degenerate Higher-Order Scalar Tensor Theories

(also called EST, Extended Scalar-Tensor)

Lagrangian is degenerate = different DOF counting

- Vector-tensor
- Scalar-Vector-Tensor (STeVe)
- ……..

Cosmological parameterizations

Too many theories, not enough data!

Can parameterize cosmology on linear scales:

Horndeski: α_M α_K α_B $\alpha_T = c_T^2 - c^2$

$\dot{G}/HG$ $(\partial\phi)^n$ $(\partial\phi)^2\Box\phi$ speed of tensors

beyond Horndeski: α_H

free functions of time

DHOST: α_V β_1 β_2 β_3

Bellini & Sawicki '14

Black holes

No hair theorem!

(Hui & Nicolas '14)

Assumptions:

- Asymptotic flatness
- Shift-symmetry

Gauss-Bonnet

Loop-hole:

(Sotiriou & Zhao '14)

Subsector equivalent to $f(\phi)\mathcal{G}$

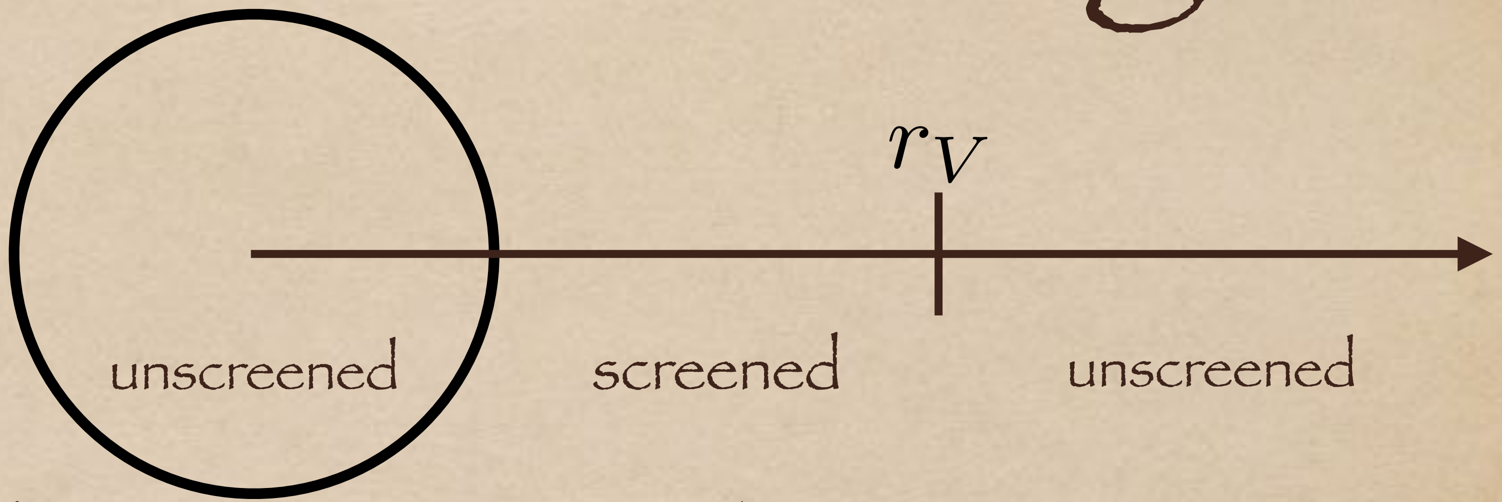
Break assumptions

Lots of examples:

- Asymptotic (A)dS
- "Stealth solutions" (Schwarzhild + non-trivial scalar)
- $\phi(t,r) = qt + \psi(r)$ (is this physical?)
- Kerr or spinning?
- Fab four (solves CC problem but too well!)

9) Vainshtein breaking

Vainshtein breaking



- IF dark energy is beyond Horndeski or DHSOT (I will stick to BH)
- Then Vainshtein mechanism is broken
- Works outside objects
- Doesn't work inside objects
- Can test cosmology on small scales!

Vainshtein breaking

$$\mathrm{d}s^2 = (-1 + 2\Phi)\mathrm{d}t^2 + (1 + 2\Psi)\,\delta_{ij}$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM}{r^2} + \frac{\Upsilon_1}{4}\frac{\mathrm{d}^2 M}{\mathrm{d}r^2}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{GM}{r^2} - \frac{5\Upsilon_2}{4r}\frac{\mathrm{d}M}{\mathrm{d}r}$$

$\gamma = 1$

unscreened

This can be used to test cosmology

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM}{r^2} + \frac{\Upsilon_1}{4}\frac{\mathrm{d}^2 M}{\mathrm{d}r^2}$$

← stellar structure, rotation curves

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{GM}{r^2} - \frac{5\Upsilon_2}{4r}\frac{\mathrm{d}M}{\mathrm{d}r}$$

← Lensing

$$\Upsilon_1 = \frac{4\alpha_H^2}{c_T^2(1+\alpha_B) - \alpha_H - 1}$$

$$\Upsilon_2 = \frac{4\alpha_H(\alpha_H - \alpha_B)}{5\left[c_T^2(1+\alpha_B) - \alpha_H - 1\right]}$$

} Can directly probe cosmology

Neutron star tests

Vainshtein breaking also works in neutron stars

- Vainshtein breaking depends on cosmology
- Need to calculate this to have relativistic system
- Calculation is Model and background-dependent!
- Will use de Sitter background for simplicity

Step 1: write down a model

canonical scalar+EH+CC

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[M_{\mathrm{pl}}^2\left(R-2\Lambda\right) - k_2\nabla_\mu\phi\nabla^\mu\phi\right.$$
$$\left. + f_4\left(2\nabla^\mu\phi\nabla^\nu\phi\left[\nabla_\mu\nabla_\nu\phi\Box\phi - \nabla_\mu\nabla_\sigma\phi\nabla^\sigma\nabla_\nu\phi\right] + \frac{X}{2}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\right)\right]$$

beyond Honrdeski quartic galilen

(simplest model we found!)

Step 2: find dS solution

need static slicing

$$\mathrm{d}s^2 = -e^{\nu(r)}\mathrm{d}t^2 + e^{\lambda(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2$$

$$\nu(r) = -\lambda(r) = \ln\left(1 - H^2r^2\right)$$

$$\phi = v_0 t + \frac{v_0}{2H}\ln\left(1 - H^2r^2\right)$$

$$k_2 = -2\frac{M_{\rm pl}^2 H^2}{v_0^2}\left(1 - \frac{\Lambda}{3H^2M_{\rm pl}^2}\right)$$

$$f_4 = \frac{M_{\rm pl}^2}{6}v_0^4\left(1 - \frac{\Lambda}{3H^2M_{\rm pl}^2}\right)$$

} Model parameters give cosmology

Step 3: add matter

$$T^{\mu}_{\nu} = \mathrm{diag}(-\varepsilon(r)\,, P(r)\,, P(r)\,, P(r))$$

perturbations sourced by star

$$\nu(r) = \ln\left(1 - H^2 r^2\right) + \delta\nu(r)$$

$$\lambda(r) = -\ln\left(1 - H^2 r^2\right) + \delta\lambda(r)$$

$$\phi = v_0 t + \frac{v_0}{2H} \ln\left(1 - H^2 r^2\right) + \delta\phi(r)$$

Step 4: match to weak field

a) Derive equations — long, needs xAct

b) Take sub-horizon limit — needs Mathematica

c) Take weak-field limit

d) Determine G and by matching to

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM}{r^2} + \frac{\Upsilon_1}{4}\frac{\mathrm{d}^2 M}{\mathrm{d}r^2}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{GM}{r^2} - \frac{5\Upsilon_2}{4r}\frac{\mathrm{d}M}{\mathrm{d}r}$$

Bonus:

$$G \rightarrow \frac{3}{5}G\left(5\frac{\Lambda}{3H^2M_{\mathrm{pl}}^2} - 2\right)^{-1}$$

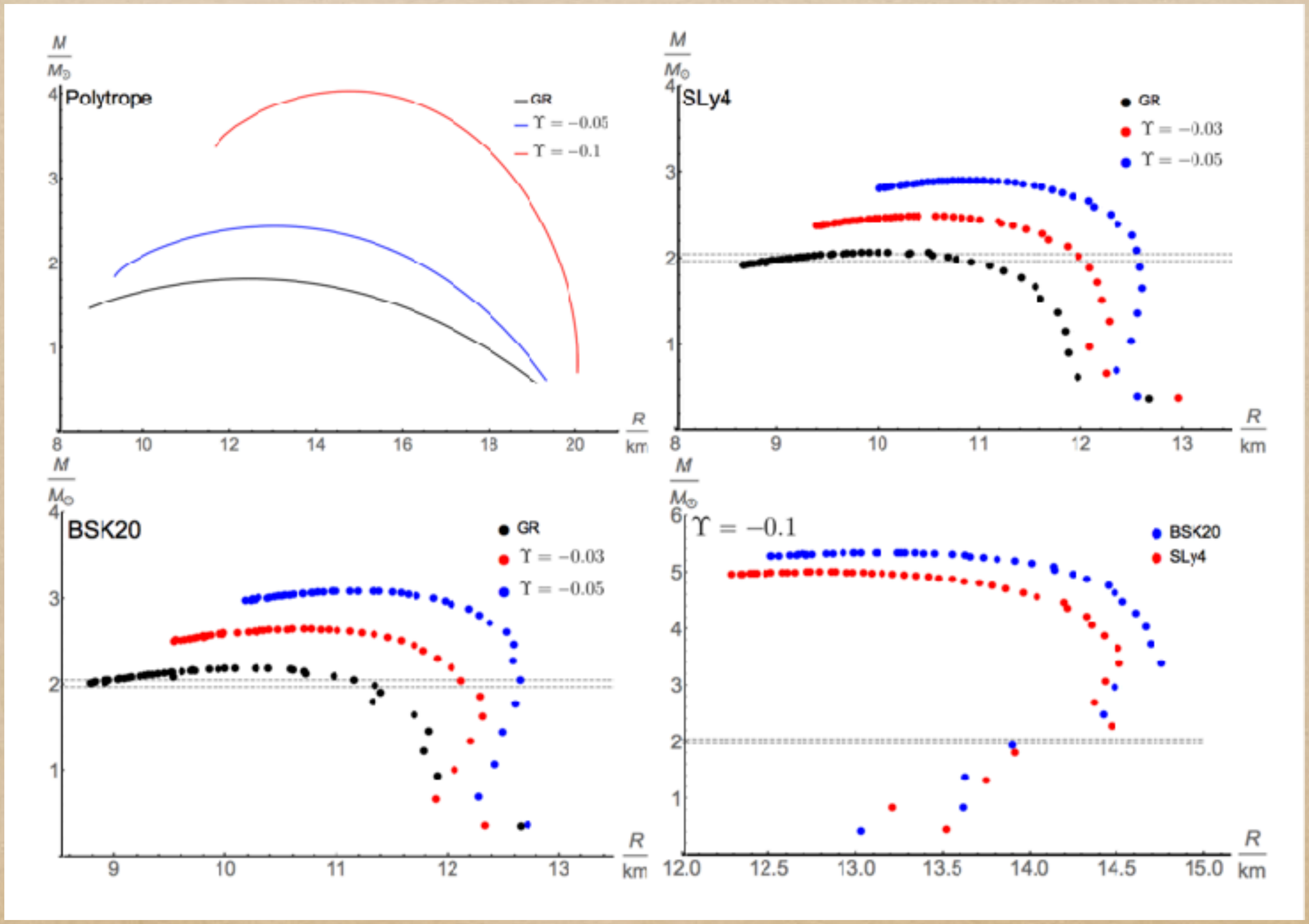
Solution in exterior is <u>EXACT</u> Schwarzschild-de Sitter

This implies

$$\Upsilon_1 = \Upsilon_2 = \beta = \frac{1}{3}\left(1 - \frac{\Lambda}{3H^2M_{\mathrm{pl}}^2}\right)$$

Note: this is a property of the model, not general

Step 5: use this to find TOV



Drawbacks

- Model-dependent
- Only works for exact de Sitter (no static FRW slicing)
- Intensive (de Sitter, sub-horizon, weak field, TOV)
- Hard to get PPN parameters

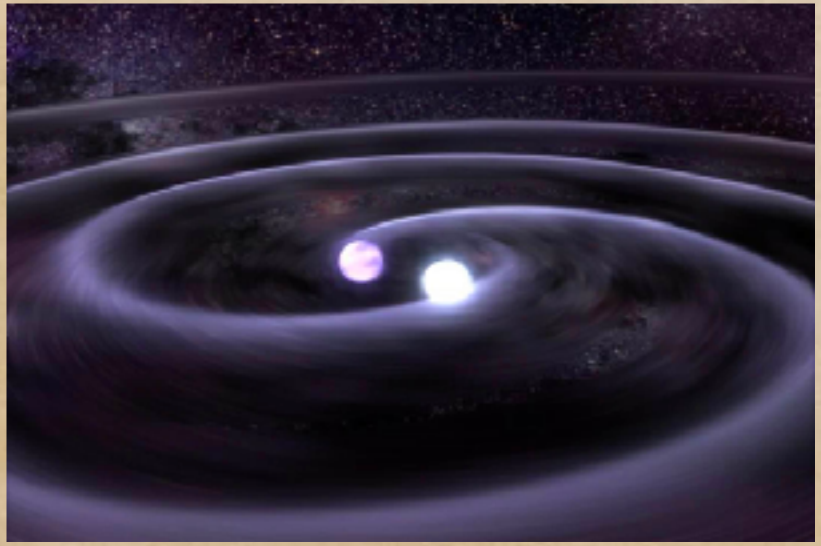
  (unless exact Schwarzchild-de Sitter)

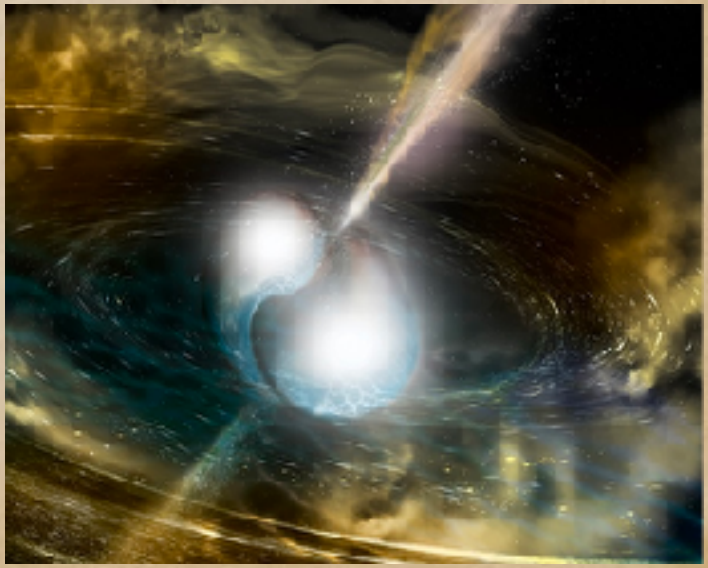
Future idea: Fermi-normal coordinates

$$\mathrm{d}s^2 = -\left[1 - (\dot{H} + H^2)|x|^2\right]\mathrm{d}t^2 + \left[1 - H^2|x|^2\right]\mathrm{d}\vec{x}^2$$

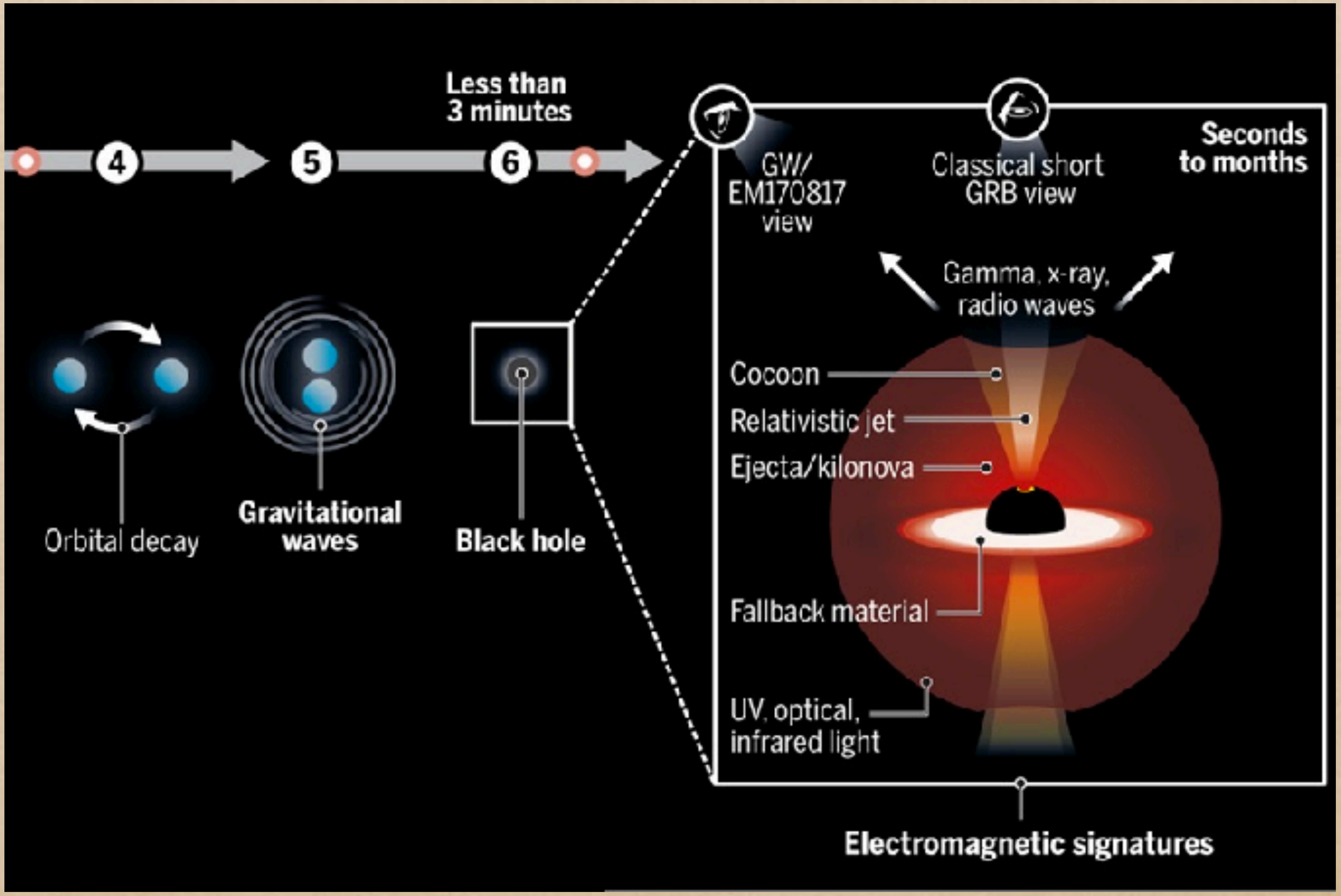
- Sub-horizon limit of FRW
- Perturbation of Minkowski
- Not unique (so what?)
- Could do PPN!

10) Cosmology after GW170817





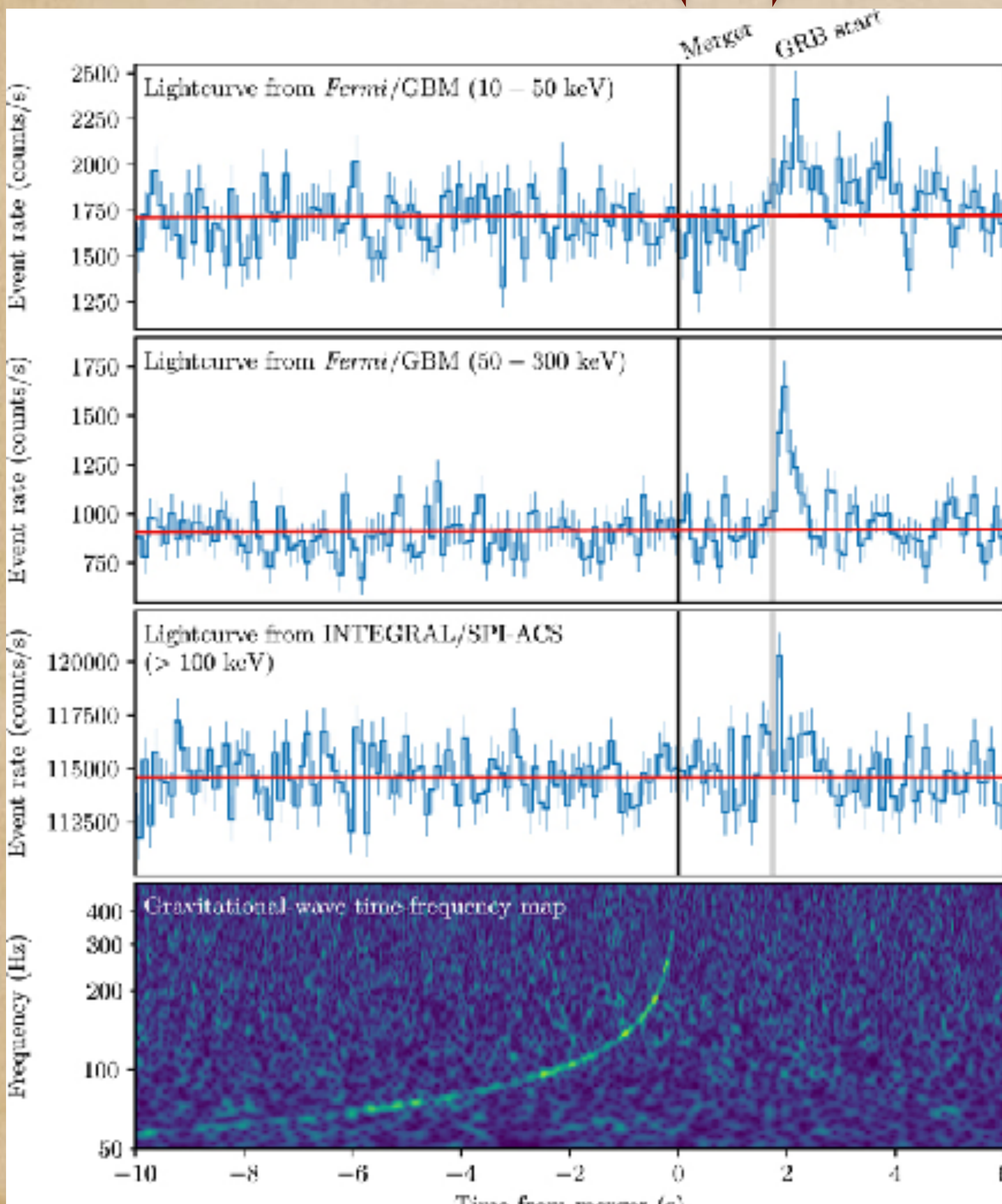
What was GW170817?



What did we measure?

GW170817

$\Delta t \leq 1.7s$



uncertainty due to NS physics

different models for GRB production

Geometry of GW170817

40 Mpc

$$\Rightarrow \frac{c_T - c}{c} < 10^{-15}$$

Constraints on new DOF

Horndeski:

$c_T = c$

$$\frac{\mathcal{L}}{\sqrt{-g}} = K(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$

$$+G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$$

$$-\frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\Box\phi + 2\nabla^\nu\nabla_\mu\phi + \nabla^\alpha\nabla_\nu\phi\nabla^\mu\nabla_\alpha\phi\right]$$

$c_T \neq c$

Strong constraints on: beyond Horndeski, DHOST, vector-tensor

JS & Jain '17, Baker et al. '17, Ezquiaga & Zumalacárregui '17, Creminelli & Vernizzi '17

Implications for DE

$$\frac{\mathcal{L}}{\sqrt{-g}} = K(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi)R$$

quintessence
K-essence
(DE not MG)

can get acceleration
strongly disfavored (7σ)
(Renk et al. '17)

no self-acceleration

preserves light-cone

Other theories: $g_{\mu\nu} = \Omega^2(\phi, X)\tilde{g}_{\mu\nu}$

(can tune functions too)

Baker et al. '17, Ezquiaga & Zumalacárregui '17

What about strong field?

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/$f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)$·Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

What does ruled out mean?

Ruled out as dark energy!

What about strong field?

Horndeski + beyond is an effective field theory:

cut-off if DE is: $\Lambda_3 = (M_{\rm pl} H_0^2)^{\frac{1}{3}} \sim (1000 \text{ km})^{-1}$

different if not DE

can't trust theories on distances smaller than this

Horndeski DE is an IR-modification of gravity!
(modifies on large distances, low energies, low cut-off)

Strong field is fine!

Use Horndeski theory as a UV-modification of gravity!
(large cut-off, important on small scales, high energies)

Features:

- Asymptotic flatness is fine
- Vainshtein breaking with $\Upsilon_i \ll 1$ ($\mathcal{O}(1)$ for DE)
- Gauss-Bonnet is not ruled out!

Vainshtein breaking tests cosmology!

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM}{r^2} + \frac{\Upsilon_1}{4}\frac{\mathrm{d}^2 M}{\mathrm{d}r^2}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{GM}{r^2} - \frac{5\Upsilon_2}{4r}\frac{\mathrm{d}M}{\mathrm{d}r}$$

$$\Upsilon_1 = \frac{4\alpha_H^2}{c_T^2(1+\alpha_B) - \alpha_H - 1}$$

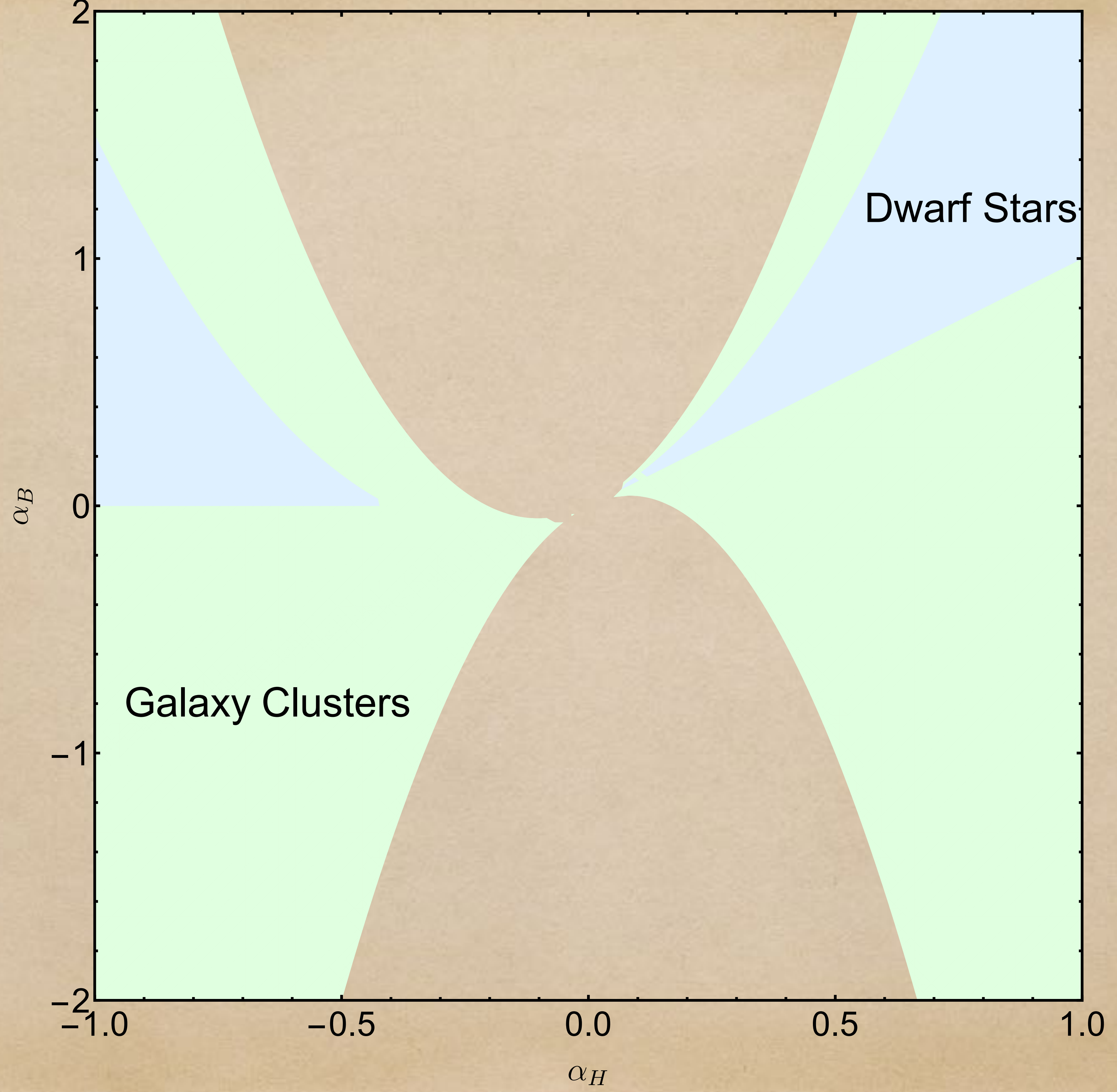
$$\Upsilon_2 = \frac{4\alpha_H(\alpha_H - \alpha_B)}{5[c_T^2(1+\alpha_B) - \alpha_H - 1]}$$

$c_T \approx 1$

breaks degeneracy

Vainshtein breaking

measures α's



2
1
0
−1
−2
−1.0
−0.5
0.0
0.5
1.0
α_B
α_H
Dwarf Stars
Galaxy Clusters

JS & Jain '17

End of lecture 1

Take away messages:

- The universe is accelerating!
- Screening mechanisms needed or theory is boring
- Chameleon, Symmetron: no hair, show up in NR regime
  (PPN harder than you think!)
- Vainshtein: non-linear, no hair
- Vainshtein breaking: neutron star tests?
- Horndeski + alive for strong field